Forecasting S&P 100 Volatility : The Incremental Information Content of Implied Volatilities and High Frequency Index Returns

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Abstract

The information content of implied volatilities and intraday returns is compared, in the context of forecasting index volatility over horizons from one to twenty days. Forecasts of two measures of realised volatility are obtained after estimating ARCH models using daily index returns, daily observations of the VIX index of implied volatility and sums of squares of five-minute index returns. The in-sample estimates show that nearly all relevant information is provided by the VIX index and hence there is not much incremental information in high-frequency index returns. For out-of-sample forecasting, the VIX index provides the most accurate forecasts for all forecast horizons and performance measures considered. The evidence for incremental forecasting information in intraday returns is insignificant.

JEL Classification: C22; C53; G13; G14

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1. Introduction

The ability of ARCH models to provide good estimates of equity return volatility is well documented. Many studies show that the parameters of a variety of different ARCH models are highly significant in-sample; see Bollerslev (1987), Nelson (1991), Glosten, Jagannathan and Runkle (1993) and surveys by Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993) and Bollerslev, Engle and Nelson (1994).

There is less evidence that ARCH models provide good forecasts of equity return volatility. Some studies (Akgiray (1989), Heynen and Kat (1994), Franses and Van Dijk (1995), Brailsford and Faff (1996), Figlewski (1997)) examine the out-of-sample predictive ability of ARCH models, with mixed results. All find that a regression of realised volatility on forecast volatility produces a low R^2 statistic (often less than 10 %) and hence the predictive power of the forecasts may be questionable. Dimson and Marsh (1990) show that data snooping can produce enhanced in-sample predictive ability that is not transferable to out-of-sample forecasting. Nelson (1992) uses theoretical methods to show that the predictive ability of ARCH models is very good at high frequencies, even when the model is misspecified, but that out-of-sample forecasting of medium to long term volatility can be poor. Nelson and Foster (1995) develop conditions for ARCH models to perform favourably at medium to long term forecast horizons.

An alternative to ARCH volatility forecasts is to use implied volatilities from options. These are known to covary with realised volatility (Latane and Rendleman (1976), Chiras and Manaster (1978)) and hence it is of interest to compare the forecasting accuracy of ARCH with implied volatility. To do this many studies use S&P 100 index options which are American. Day and Lewis (1992) find that implied volatilities perform as well but no better than forecasts from ARCH models. Mixtures of the two forecasts outperform both univariate forecasts. Canina and Figlewski (1993) provide contrary evidence to Day and Lewis. They find that implied volatilities are poor forecasts of volatility and that simple historical volatilities outperform implied volatilities. Christensen and Prabhala (1998) show for a much longer period that while implied volatilities are biased forecasts of volatility they outperform historical information models when forecasting volatility.

These papers use implied volatilities that contain measurement errors, because early exercise is ignored and/or dividends are ignored and/or the spot index contains stale prices. The implied volatility series are potentially flawed and this may account for some of the conflicting results. Fleming, Ostdiek and Whaley (1995) describe an implied volatility index (*VIX*) which eliminates mis-specification problems. We use the same index to obtain new results about the information content of option prices. Fleming (1998) uses a volatility measure similar to *VIX* to show that implieds outperform historical information.

From the recent studies of Christensen and Prabhala (1998) and Fleming (1998) it can be concluded that the evidence now favours the conclusion that implied volatilities are more informative than daily returns when forecasting equity volatility. The same conclusion has been obtained for foreign exchange volatility from daily data, but with more certainty (Jorion (1995), Xu and Taylor (1995)). High-frequency returns, however, have the potential to change these conclusions. Taylor and Xu (1997) show that five-minute FX returns contain volatility information incremental to that provided by options. In this paper we explore the incremental volatility information of high-frequency stock index returns for the first time.

The importance of intraday returns for measuring realised volatility is demonstrated for FX data by Taylor and Xu (1997), Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold and Labys (2000), and for equities by Ebens (1999) and Andersen, Bollerslev, Diebold and Ebens (2000). Andersen and Bollerslev (1998) prove that regression methods will give low R^2 values when daily squared returns measure realised volatility, even for optimal GARCH forecasts, because squared returns are noisy estimates of volatility. They show that intraday returns can be used to construct a realised volatility series that essentially eliminates the noise in measurements of daily volatility. They find remarkable improvements in the forecasting performance of ARCH models for FX data when they are used to forecast the new realised series, compatible with theoretical analysis. They do not, however, compare ARCH forecasts with implied forecasts.

This paper answers some important empirical questions for the S & P 100 index. Firstly, how does the predictive quality of volatility forecasts from ARCH models, that use daily index returns and/or intraday returns, compare with forecasts from models that use information contained in implied volatilities? Secondly, how important is the selection of the measure of realised volatility in assessing the predictive accuracy of volatility forecasts?

The paper is arranged as follows. Section two discusses the various datasets we use and issues surrounding their construction. Methods for estimating and forecasting volatility that use daily index returns, five-minute returns and implied volatilities are presented in Section three. Section four provides results from both in-sample estimation and out of sample forecasting of S & P 100 volatility. Section five sets out our conclusions.

2. Data

There are three main types of data: daily index returns, daily implied volatilities and five-minute index returns. These are available to us for the thirteen years from 1987 to 1999 inclusive. The in-sample period is from 2 January 1987 to 31 December 1992 providing 1,519 daily observations, followed by the out-of-sample period from 4 January 1993 to 31 December 1999 providing 1,768 daily observations.[1]

2.1. Daily index returns

Daily returns from the S&P 100 index are defined in the standard way by the natural logarithm of the ratio of consecutive daily closing levels. Index returns are not adjusted for dividends. Some models in this paper have been estimated using dividend adjusted index returns and the results from volatility estimation and forecasting are not significantly different.

2.2. Implied volatilities

Implied volatilities are considered to be the market's forecast of the volatility of the underlying asset of an option. To calculate an implied volatility an option valuation model is needed as well as inputs for that model (index level, risk free rate, dividends, contractual provisions of the option) and an observed option price. Many of these variables are subject to measurement errors that may induce biases in a series of implied volatilities.

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[1] An earlier version of this paper used a much shorter out-of-sample period.

The use of an inappropriate option valuation model will also cause mis-measurements in the implied volatilities. For instance, S&P 100 options are American but if an European model is used to calculate implied volatilities then error will be induced by the omission of the early exercise option. Moreover, when a valuation model is used that includes the early exercise option, if the timing and level of dividends are assumed constant rather than using actual timings and dividends then the early exercise option is not taken up as frequently, pricing errors will occur, and implied volatilities will be mis-measured (see Harvey and Whaley (1992)).

Important biases may also be induced by relatively infrequent trading of the stocks in the index (see Jorion (1995)). Also, if closing prices are used there may be biases due to the different closing times of stock and options markets. Finally if bid or ask option prices are used the first order autocorrelation of the implied volatility series may be negative and so biases will result from bid/ask effects.

Implied volatilities used in many previous studies (including Day and Lewis (1992), Canina and Figlewski (1993) and Christensen and Prabhala (1998)) contain relevant measurement errors whose magnitudes are unknown. We follow Fleming, Ostdiek and Whaley (1995) and use an implied volatility index (*VIX*) that mitigates the above problems. *VIX* is a weighted index of American implied volatilities calculated from eight near-the-money, near-to-expiry, S & P 100 call and put options and it is constructed in such a way as to eliminate mis-measurement and “smile” effects. This makes it a more accurate measurement of implied market volatility. *VIX* uses the binomial valuation method with trees that are adjusted to reflect the actual amount and timing of anticipated cash dividends. Each option price is calculated using the midpoint of the most recent bid/ask quote to avoid problems due to bid/ask bounce. *VIX* uses both call and put options to increase the amount of information and, most important, to eliminate problems due to mis-measurement of the underlying index and put/call option clientele effects.

VIX is constructed so that it represents a hypothetical option that is at-the-money and has a constant 22 trading days (30 calendar days) to expiry. *VIX* uses pairs of near-the-money exercise prices, that are just above the current index price and just below it. *VIX* also uses pairs of times to expiry, one that is nearby (at least 8 calendar days to expiration) and one that is second nearby (the following contract month). For a detailed explanation of the construction of the *VIX* index see Fleming, Ostdiek and Whaley (1995).

Because *VIX* eliminates most of the problems of mis-measurement we use it as our measure for S&P 100 index implied volatility. Daily values of *VIX* at the close of option trading are used[2]. Even though *VIX* is robust to mis-measurement it is still a biased predictor of subsequent volatility. It is our understanding that bias occurs because of a trading time adjustment that typically multiplies[3] conventional implied volatilities by approximately 1.2. The variance of daily index returns from 1987 to 1999 is equivalent to an annualised volatility or standard deviation of 17.6%. The average value of *VIX* squared is equivalent to an annualised volatility of 21.7% during the same period, which is consistent with an average scaling factor of 1.23. With this in mind, our empirical methods are designed to be robust against this bias.

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[2] *VIX* data are available at http://www.cboe.com/tools/historical/vix.htm.

[3] *VIX* multiplies the conventional implied by $\sqrt{N_c/N_t}$, with N_c and N_t respectively calendar days and trading days until expiry. See Fleming et al (1995), equation 2 and Figure 3.

2.3. High Frequency Stock Returns

ARCH and implied volatility models provide forecasts of volatility. These forecasts require some measure of the latent volatility process so that their performance can be evaluated. Because the volatility process is not observed researchers have used a variety of methods to compute ex post estimates of volatility, often called realised volatility. The most common method for computing a realised volatility is to square the inter-period returns. For example, if we are forecasting daily volatility the realised measure is the squared daily return. Andersen and Bollerslev (1998) show that this method produces inaccurate forecasts for correctly specified volatility models. Consider a representation of excess returns r_t such that $r_t = s_t z_t$ where s_t is the time-varying volatility and $z_t \sim \text{i.i.d.}(0,1)$. The realised volatility measure using squared returns is $r_t^2 = s_t^2 z_t^2$ and if s_t is independent of z_t then $E\left(r_t^2 \middle| s_t^2\right) = s_t^2$. However the realised measure, r_t^2, will be a very noisy estimate of s_t^2 because not only are ARCH models generally misspecified but poor predictions of r_t^2 will be due to the noisy component z_t^2.

By sampling more frequently and producing a measure based on intraday data, Andersen and Bollerslev (1998) show that the noisy component is diminished and that in theory the realised volatility is then much closer to the volatility during a day. They find, for foreign exchange data, that the performance of ARCH models improves as the amount of intraperiod data used to measure realised volatility increases. Related results for FX and equity data are presented respectively in Andersen, Bollerslev, Diebold and Labys (2000) and Andersen, Bollerslev, Diebold and Ebens (2000).

In this paper we use five-minute returns from the S&P 100 index to calculate a measure of realised volatility, here called *INTRA*. These five-minute returns are constructed from the contemporaneous index levels recorded when S&P 100 options are traded. The original dataset contains an index level whenever an option is traded but we have chosen a five-minute frequency as this is the highest and best frequency that Andersen and Bollerslev (1998) use. The latest index level available before a five-minute mark is used in the calculation of five-minute returns. To construct the measure *INTRA* of daily realised volatility we square and then sum five-minute returns for the period from 08:30 CST until 15:00 CST and then add the square of the previous "overnight" return. Thus *INTRA* on a Tuesday, for example, is the squared index return from Monday 15:00 to Tuesday 08:30 plus 78 squared five-minute index returns on Tuesday, commencing with the return from 08:30 to 08:35 and concluding with the return from 14:55 to 15:00. This volatility measure is also used in ARCH models to deduce the information content of intraday returns.

The square root of the average value of *INTRA* from 1987 to 1999 is equivalent to an annualised volatility of 13.4%. This is much less than the equivalent volatility calculated from the variance of daily returns for the same period, namely 17.6%. This is a consequence of substantial positive correlation between consecutive intraday returns, which could be explained by stale prices in the S & P 100 index.

3. Methodology for forecasting volatility

3.1. In-sample models

To compare the in-sample performance of several models that use information from stock index returns and implied volatilities, ARCH models are estimated for daily index returns r_t from 2 January 1987 to 31 December 1992, with a dummy term d_t for the Black Monday crash in 1987. The most general specification is as follows :

$$r_t = m + y_1 d_t + e_t , \tag{1}$$

$$e_t = h_t^{\frac{1}{2}} z_t , \qquad z_t \sim \text{i.i.d.}(0,1), \tag{2}$$

$$h_t = \frac{a_0 + a_1 e_{t-1}^2 + a_2 s_{t-1} e_{t-1}^2 + y_2 d_{t-1}}{1 - bL} + \frac{gINTRA_{t-1}}{1 - b_I L} + \frac{dVIX_{t-1}^2}{1 - b_V L} . \tag{3}$$

Here L is the lag operator, h_t is the conditional variance of the return in period t, s_{t-1} is 1 when $e_{t-1} < 0$ and otherwise it is zero, d_t is 1 when t refers to 19 October 1987 and is 0 otherwise, VIX_{t-1} is the daily implied index volatility computed from an annualised volatility as $VIX/\sqrt{252}$ and $INTRA_{t-1}$ is the measure of volatility based on five-minute returns from the S&P 100 index. The parameters y_1 and y_2 respectively represent the excess return on the crash day (-24%) and an exceptional, transient effect in variance immediately following the crash.

By placing restrictions on certain parameters six different volatility models are obtained using different daily information sets :

1. The GJR(1,1) model of Glosten, Jagannathan and Runkle (1993) with a crash dummy in the variance equation, also estimated by Blair, Poon and Taylor (2000, 2001) : $g = d = b_I = b_V = 0$.
2. A volatility model that uses intraday returns information alone : $a_1 = a_2 = d = b_V = 0$.
3. A specification that uses the historic information in both daily returns and *INTRA* but ignores option information : $d = b_V = 0$.
4. A volatility model that uses the latest information in the *VIX* series alone : $a_1 = a_2 = g = b_I = 0$.

5. A low-frequency specification that only uses the information provided by daily measurements of *VIX* and index returns : $g = b_I = 0$.
6. A volatility model that uses information in both *INTRA* and *VIX* without explicit use of daily returns : $a_1 = a_2 = 0$.

These are all special cases of :

7. The unrestricted model using all the available information in daily returns, *INTRA* and *VIX*.

The parameters are estimated by the usual quasi-likelihood methodology[4], so that the likelihood function is defined by assuming standardised returns, z_t, have Normal distributions even though this assumption is known to be false. Inferences are made using robust *p*-values, based on Bollerslev and Wooldridge (1992) standard errors, that are robust against mis-specification of the shape of the conditional distributions. It is then possible to attempt to decide which types of information are required to obtain a satisfactory description of the conditional distributions of daily returns. To assess predictive power, e_t^2 is regressed on the conditional variance h_t. Higher values of the correlation *R* indicate more accurate in-sample forecasts of e_t^2.

3.2. Forecasting methods

Time series of forecasts are obtained by estimating rolling ARCH models. Each model is estimated initially over the final 1,000 trading days of the in-sample period, from 20 January 1989 to 31 December 1992, and forecasts of

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[4] The general parameter vector is $(m, a_0, a_1, a_2, b, y_1, y_2, g, b_I, d, b_V)$. The log-likelihood function was maximised using GAUSS with the constraints $a_1, a_1 + a_2 \geq 0$.

realised volatility are made for the next day, say day $T+1$, using the in-sample parameter estimates. The model and data are then rolled forward one day, deleting the observation(s) at time $T-999$ and adding on the observation(s) at time $T+1$, re-estimated and a forecast is made for time $T+2$. This rolling method is repeated until the end of the out-of-sample forecast period. The one-step-ahead forecasts provide predictions for 4 January 1993 to 31 December 1999 inclusive and define time series of length 1,768. On each day, forecasts are also made for 5, 10 and 20 day volatility.

Two measures of realised volatility are predicted. The first measure is squared excess returns, so that forecasts are made at time T of

$$(r_{T+1}-m)^2 \text{ and } \sum_{j=1}^{N}(r_{T+j}-m)^2, \quad N=5,10,20.$$

These quantities are predicted because a correctly specified ARCH model provides optimal forecasts of $(r_{T+1}-m)^2$. We assume the conditional expected return m is constant and our results are not sensitive to the choice of m; results are reported for an assumed annual expected return of 10%. The second measure of realised volatility is *INTRA* that uses five-minute and "overnight" returns. Forecasts are produced at time T for

$$INTRA_{T+1} \text{ and } \sum_{j=1}^{N} INTRA_{T+j}, \quad N=5,10,20.$$

These quantities can be expected to provide a more predictable measure of realised volatility than squared excess daily returns, although on average $INTRA_{T+1}$ is less than $(r_{T+1}-m)^2$ as already remarked in Section 2.3. Four different sets of forecasts are estimated for each measure of realised volatility to compare predictive power. These forecasts are first presented for the realised measure defined by squared excess returns.

a. Historic volatility

A simple estimate of volatility is evaluated to see what simple methods can achieve in comparison with more sophisticated methodologies. The simple one-step-ahead forecast is here called historic volatility (*HV*) and it equals the sample variance of daily returns over the period from time $T-99$ to time T inclusive, given by

$$h_{T+1} = \frac{1}{100} \sum_{j=0}^{99} \left(r_{T-j} - \bar{r}_T \right)^2, \quad \bar{r}_T = \frac{1}{100} \sum_{j=0}^{99} r_{T-j}. \tag{4}$$

To produce 5, 10 and 20 day volatility forecasts the one-step-ahead forecast is respectively multiplied by 5, 10 and 20.

b. GJR(1,1) forecasts

The one-step-ahead forecast, h_{T+1}, of $(r_{T+1} - m)^2$ is defined by the recursive formula,

$$h_{t+1} = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 s_t r_t^2 + \beta\, h_t \tag{5}$$

where s_t equals 1 when $r_t < 0$ and otherwise equals 0. The parameters $\alpha_0, \alpha_1, \alpha_2, \beta$ are estimated from the information $I_T^{(1)} = \{ r_{T-i}, 0 \le i \le 999 \}$. Forecasts for 5, 10 and 20 day volatility are produced by aggregating expectations

$$E\left(h_{T+j} \middle| I_T^{(1)} \right) = \alpha_0 + p_{gjr} E\left(h_{T+j-1} \middle| I_T^{(1)} \right), \quad j > 1, \tag{6}$$

with p_{gjr} the persistence equal to $\alpha_1 + \frac{1}{2}\alpha_2 + \beta$ for the GJR(1,1) model, here assuming that returns have symmetric distributions. Then $\sum_{j=1}^{N} \left(r_{T+j} - m \right)^2$ is forecast to be $\sum_{j=1}^{N} E\left(h_{T+j} \middle| I_T^{(1)} \right)$.

c. Forecasts of volatility using VIX

These forecasts use information contained in *VIX* alone to forecast volatility. The one-step-ahead forecast, h_{T+1}, is obtained by estimating an ARCH model with conditional variances defined by

$$h_t = a_0 + \frac{dVIX_{t-1}^2}{1 - b_V L}. \tag{7}$$

Parameters a_0, d, b_V are estimated from $I_T^{(2)} = \{r_{T-i}, VIX_{T-i}, 0 \le i \le 999\}$. To produce 5, 10 and 20 day volatility forecasts it is assumed that $E\left(h_{T+j} \middle| I_T^{(2)}\right) = E\left(h_{T+j-1} \middle| I_T^{(2)}\right)$ for $j > 1$ so that the one-step-ahead forecast is respectively multiplied by 5, 10 and 20. This simple multiplicative method may handicap *VIX* forecasts, however information about the term structure of implied volatility is not available from *VIX*.

d. Forecasts of volatility using intraday returns

These forecasts use the information in intraday returns alone to forecast volatility. Now h_{T+1} is obtained from conditional variances defined by

$$h_t = a_0 + \frac{gINTRA_{t-1}}{1 - b_I L} \tag{8}$$

and the parameters a_0, g, b_I are estimated[5] from $I_T^{(3)} = \{r_{T-i}, INTRA_{T-i}, 0 \le i \le 999\}$. To produce 5, 10 and 20 day volatility forecasts, we aggregate expectations defined by

$$E\left(h_{T+j} \middle| I_T^{(3)}\right) = (1 - b_I)a_0 + b_I E\left(h_{T+j-1} \middle| I_T^{(3)}\right) + gE\left(INTRA_{T+j-1} \middle| I_T^{(3)}\right), \quad j > 1.$$

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[5] Specifications (5), (7) and (8) are estimated with the constraints $a_1, a_1 + a_2, b, b_I, b_V \ge 0$.

To make progress, it is necessary to assume that there is a proportional relationship between expectations of squared excess returns and *INTRA*, so that

$$E\left(INTRA_{T+j}\middle|I_T^{(k)}\right)=c_T\,E\left(\left(r_{T+j}-m\right)^2\middle|I_T^{(k)}\right)=c_T\,E\left(h_{T+j}\middle|I_T^{(k)}\right) \qquad (9)$$

for some constant c_T, that is the same for all positive *j* and information sets indexed by $k=1,2,3.$ Positive dependence among intraday returns causes c_T to be less than one as noted in Section 2.3. Making the assumption stated by equation (9), expectations are calculated from

$$E\left(h_{T+j}\middle|I_T^{(3)}\right)=\left(1-b_I\right)a_0+\left(b_I+c_T g\right)b_I\,E\left(h_{T+j-1}\middle|I_T^{(3)}\right),\quad j>1, \qquad (10)$$

and $\sum_{j=1}^{N}\left(r_{T+j}-m\right)^2$ is forecast to be $\sum_{j=1}^{N}E\left(h_{T+j}\middle|I_T^{(3)}\right)$. To calculate the forecasts at time *T*, the constant c_T is estimated by the ratio

$$\hat{c}_T=\sum_{t=T-999}^{T}INTRA_t\Big/\sum_{t=T-999}^{T}\left(r_t-m\right)^2. \qquad (11)$$

e. Forecasts of INTRA

To predict future values of *INTRA* we again make the plausible assumption stated in equation (9). Then if some method (*HV*, *GJR*, *VIX*, or *INTRA*) produces a forecast $f_{T,N}$ of $\sum_{j=1}^{N}\left(r_{T+j}-m\right)^2$ then that method's forecast of $\sum_{j=1}^{N}INTRA_{T+j}$ equals $\hat{c}_T\,f_{T,N}$.

3.3. Forecast evaluation

Mean square forecast errors are compared to assess the relative predictive accuracy of the four forecasting methods. Given forecasts $x_{T,N}$ made at times

$T = s,...,n-N$ of quantities $y_{T,N}$ known at time $T+N$, we report the proportion of variance explained by the forecasts :

$$P = 1 - \frac{\sum_{T=s}^{n-N}\left(y_{T,N} - x_{T,N}\right)^2}{\sum_{T=s}^{n-N}\left(y_{T,N} - \bar{y}\right)^2}. \tag{12}$$

We also report the squared correlation, R^2, from the regression

$$y_{T,N} = a + b\ x_{T,N} + u_{T,N}$$

which provides the proportion of variance explained by the ex post best linear combination $a + b\ x_{T,N}$. Hence *P*, which measures the accuracy of forecasts, is at most equal to the popular R^2 statistic that is often interpreted as a measure of information content.

The above regression only measures the predictive power of a single method in isolation. Day and Lewis (1992) found that mixtures of forecasts from ARCH models and from implied volatilities produced higher adjusted R^2 statistics than univariate forecasts. Consequently multiple regressions are also estimated. Multiple R^2 statistics are used to assess the information content of the mixtures. No model that uses a mixture of the VIX, GJR and intraday information sets is used to produce forecasts directly. Rather bivariate and trivariate mixtures of forecasts are evaluated to see if the implied volatility forecasts subsume the information in GJR and intraday return forecasts.

4. Results

4.1. In-sample ARCH results

Table 1 presents parameter estimates, robust *t*-ratios, log-likelihoods and squared correlations R^2 for the seven ARCH models defined in Section 3.1.

The results are obtained from six years of data, from 2 January 1987 to 31 December 1992. The log-likelihoods increase monotonically across the columns of Table 1. The excess log-likelihood for a model is defined as its maximum log-likelihood minus the corresponding figure for the first model.

The first model is the standard GJR model, that uses previous index returns to describe the conditional variance, augmented by a transient volatility increase following the crash on 19 October 1987. The multiplier for squared negative returns ($a_1 + a_2$) is three times that for squared positive returns (a_1), indicating a substantial asymmetric effect, although the estimates of a_1 and a_2 are not significantly different from zero at the 10% level reflecting the relatively short sample period. The persistence estimate is $a_1 + \frac{1}{2} a_2 + b$ = 0.9693.

The second model uses only the measure *INTRA* based on five-minute returns to describe conditional variances. It has a substantially higher log-likelihood than the GJR model. The *INTRA* series is filtered by the lag function $g/(1 - b_I L)$ with b_I approximately 0.25. This suggests that most of the volatility information in five-minute returns can be obtained from the most recent day although long memory effects in *INTRA*, beyond the scope of this paper, might be anticipated from the results of Ebens (1999) and Andersen, Bollerslev, Diebold and Ebens (2000). The low value of b_I contrasts with the high value of b for daily returns (0.94 for Model 1).

When all the information in the history of daily and intraday index returns is used in the third model, the log-likelihood is not significantly higher than for Model 2 using the non-robust likelihood ratio test and a 5% significance level. Furthermore, the parameters for the variable *INTRA* are similar for Models 2 and 3 whilst the parameters for squared daily returns are much smaller for Model 3 than for Model 1.

The only information used in the fourth model to describe conditional variances is contained in the *VIX* series. The significance of this information is clear from the high robust t-ratio for the parameter d, equal to 6.31. The fourth model has a higher likelihood than the second model that uses daily and five-minute returns. Model 4 has the same number of parameters as Model 2 and the difference in their log-likelihoods is 6.17. Thus implied volatilities are more informative than five-minute returns, at least in-sample. The *VIX* series is filtered by the function $d/(1-b_V L)$ with b_V approximately 0.15. Although b_V is near zero, it is significantly different at the 5% level with a robust t-ratio of 2.73. This result suggests that the options market is not informationally efficient, since the latest option prices do not contain all relevant information about the next day's volatility. Although *VIX* quadrupled on the crash day, from 36% on Friday 16th to 150% on Monday 19th, the estimate of the crash parameter y_2 is positive.

Models 5, 6 and 7 are compared with Model 4 to try to decide if *VIX* contains all the relevant information about the next day's volatility. Any incremental information in the history of five-minute and daily returns is probably to be found in the five-minute returns, firstly because Model 6 has a higher log-likelihood than Model 5 and secondly because Model 7 that uses all the information improves only slightly on Model 6 that does not use daily returns. Consequently, we focus on comparing Model 4 that uses *VIX* information with Model 6 that uses both *VIX* and *INTRA* information.

The log-likelihood for Model 6 is 7.73 more than for Model 4. The former model has an additional two parameters and hence a non-robust likelihood-ratio test would conclusively prefer the former model. However, the robust t-ratio for the *INTRA* parameter g equals 1.86 so that g has a robust one-tailed p-value that is low (3.1%) but not very close to zero. It may be concluded that whilst *VIX* is more informative than *INTRA* there is probably incremental information in five-minute returns.

As *VIX* squared and *INTRA* do not have comparable magnitudes, for reasons given in Sections 2.2 and 2.3, the fact that the *VIX* parameter d is less than the *INTRA* parameter g is not particularly relevant when comparing information content. The average level of *VIX* squared is approximately 2.6 times the average level of *INTRA*. In the equation for the conditional variance h_t the ratio of the impact of *VIX* information to the impact of *INTRA* information is therefore approximately

$$2.6\frac{d/(1-b_V)}{g/(1-b_I)} = 2.8$$

using the parameter estimates for Model 6. This ratio emphasises that *VIX* is much more informative than *INTRA* when applying an in-sample methodology.

Figure 1 provides a comparison of the in-sample conditional variances for three ARCH specifications and the squared returns that they forecast. The specifications are Models 1, 2 and 4 that respectively use the information in daily returns (labelled *GJR* on Figure 1), intraday returns (labelled *INTRA*) and implied volatilities (labelled *VIX*). The values of the four time series are plotted on a logarithmic scale truncated at 10^{-5}. It can be seen that the squared returns are a very noisy series and that the three conditional variances move closely together.

4.2. Out of sample forecasting

The out-of-sample accuracy of volatility forecasts is compared from 4 January 1993 to 31 December 1999. Figure 2(a) provides similar information to Figure 1 and shows the one-step ahead *GJR*, *INTRA* and *VIX* forecasts compared with the realised volatility defined by squared returns. It is now possible to see periods when the three forecasts are some way apart and other periods when they are very similar. Figure 2(b) provides comparisons when there is much less noise in the series of realised volatility. The quantity predicted is now the sum of *INTRA* values over twenty days. It is seen that volatility was low for the first four years on the plot (1993-1996), higher in 1997 and then much higher in 1998 and 1999. The *GJR* forecasts are the most sensitive to large movements in the index and occasional spikes in the forecasts can be seen. The *VIX* forecasts can be seen to track the quantity forecasted more closely than the other forecasts during some periods, for example throughout 1993 at the left side of the figure. Figure 3 plots the sums of *INTRA* values over twenty days against the *VIX* forecasts, for which R^2 equals 0.57.

a. Relative accuracy of the forecasts

Tables 2 and 3 summarise the out-of-sample accuracy of the volatility forecasts. Each table provides information for forecasts 1, 5, 10 and 20 days ahead of the two measures of realised volatility. Both measures are sums of squares, summing squared daily excess returns for one measure and squared five-minute and overnight returns for the other measure. Table 2 reports the accuracy measure *P* defined by equation (12). This proportion of explained variability is a linear function of the mean square error of forecasts, with higher values corresponding to more accurate forecasts. Table 3 reports the (multiple) squared correlation R^2 from regressions of realised volatility on one or more forecasts. These R^2 statistics provide useful information about the information content of the various forecasts.

Tables 2 and 3 unequivocally support the conclusion that the implied volatility index *VIX* provides more accurate forecasts of realised volatility than the historic volatility *HV*, the low-frequency ARCH forecast *GJR* given by the model of Glosten et al (1993) and the intraday volatility *INTRA* calculated from five-minute and overnight returns. The *VIX* forecasts are more accurate than the other univariate forecasts for all 16 combinations of realised volatility measure (sums of squared excess returns or *INTRA*), forecast horizon (1, 5, 10 or 20) and summary statistic (*P* or R^2).

The proportions of explained variability *P* given in Table 2 can be summarised by five remarks. First, *VIX* always has the highest value[6]. Second, *GJR* and *INTRA* have similar values when forecasting for the next day. Third, *INTRA* ranks second to *VIX* when forecasting five or more days into the future. Fourth, the values of *P* generally increase as the horizon *N* increases, with the exception of the *GJR* forecasts. Fifth, the values of *P* are much higher for predictions of *INTRA* than for predictions of squared daily excess returns, as anticipated from the work of Andersen and Bollerslev (1998). Changing the target to be forecast from squared daily excess returns to sums of squared intraday returns approximately quadruples the proportions *P* for one-day ahead forecasts and more than doubles them for five-day ahead forecasts.

The rows of Table 3 are arranged in the order that produces columns of ranked values of R^2 for $N = 5, 10, 20$ and almost a ranked column when *N* is 1. It is seen first that the variance of the 100 previous daily returns is less informative than all other forecasts. Second, *INTRA* has a higher statistic than *GJR*, except when *N* is 1, as also occurs in Table 2. Third, linear combinations of *GJR* and *INTRA* are less informative than *VIX*. Fourth, combinations of *VIX* and other forecasts are apparently only more informative than *VIX* alone when

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[6] However, there are few differences in the values of *P* that are statistically significant at conventional levels.

forecasting one or five days day ahead using *VIX* and *GJR*. The lack of incremental information is striking for forecasts ten and twenty days ahead; the value of R^2 increases by less than 0.001 when the first realised volatility measure is regressed on *VIX*, *GJR* and *INTRA* instead of *VIX* alone, whilst the increase is from 0.559 to 0.565 for the second measure when *N* equals 10 and from 0.569 to 0.577 when *N* equals 20. The superior performance of *VIX* as the forecast horizon increases is consistent with its design as a measure of market expectations over 22 trading days. Fifth, the values of R^2 generally increase as *N* increases and the values are higher, often much higher, when realised volatility is measured using intraday returns.

The differences between the two performance measures, $R^2 - P$, are generally small. In particular, they are always less than 0.05 for the *VIX* forecasts thus indicating that a linear function of these forecasts, optimised ex post, is not much more accurate than the ex ante forecasts. However, the differences $R^2 - P$ are substantial for the *GJR* forecasts and this is a consequence of the low values of *P* when forecasting 10 or 20 days ahead.

b. Properties of univariate forecast errors

Tests for bias in the forecasts of squared excess returns find no evidence of bias, for all horizons *N* that have been considered. There is, however, statistically significant evidence that the forecasts of future *INTRA* values are downward biased for the out-of-sample period. For example, the average value of the *INTRA* forecast obtained from *VIX* information is 86% of the average value of *INTRA*, for each of the four forecasting horizons; t-tests for a zero mean applied to non-overlapping forecast errors range from 3.02 to 4.09. The observed level of bias is consistent with an overall increase in estimates $\hat{c}_T$ of the terms c_T defined by equation (9), from 0.6 in 1993 to 0.8 in 1999, that is consistent with a decrease in the dependence between consecutive intraday returns. As the estimates $\hat{c}_T$ use information from the previous four years they are, with hindsight, less than the parameters c_T that they estimate and hence the forecasts are on average less than the intraday volatility that they predict.

The values of R^2 in Table 3 imply that there is not much incremental predictive information in the *GJR* and *INTRA* forecasts, and that any such information is to be found at the shortest forecasting horizons. The increases in R^2 when *GJR* and *INTRA* are both added to the regressions that only employ *VIX* are 5.0%, 1.9%, 0.6% and 0.8% when predicting realised volatility measured by *INTRA* at horizons 1, 5, 10 and 20 respectively; the increases are much less when predicting squared excess returns and their sums. Consequently, we only discuss the incremental predictive information beyond *VIX* for one-step ahead forecasts. Let *GJRf*, *VIXf* and *INTRAf* refer to the forecasts of the next squared excess return defined by equations (5), (7) and (8) respectively. A heteroscedasticity-adjusted regression of the forecast errors from *VIX* on the three sets of forecasts then gives

$$\begin{aligned}(r_{T+1}-m)^2 - VIXf_T &= a + 0.16\,VIXf_T + 0.27\,GJRf_T \\ &\quad - 0.23\,INTRAf_T + \text{ residual}\end{aligned}$$

with standard errors 0.16, 0.13 and 0.15 for the three forecasts and a negative intercept a. The R^2 of this regression is only 0.4% with an F-statistic equal to 2.37 and a p-value of 7%. The standard errors ignore the sampling error in the parameters that are used to calculate the forecasts and hence they underestimate the correct standard errors, see West (2001). Nevertheless, it can be concluded that the evidence for incremental forecasting information in daily and intraday returns is insignificant when predicting squared daily returns.

The forecasts of $INTRA_{T+1}$ are $\hat{c}_T$ multiplied by the forecasts of the squared excess return. They provide the estimated regression

$$INTRA_{T+1} - \hat{c}_T VIXf_T = a - 0.02\,\hat{c}_T VIXf_T + 0.42\,\hat{c}_T GJRf_T - 0.12\,\hat{c}_T INTRAf_T + \text{ residual.}$$

The forecast errors now vary significantly with the forecasts. The R^2 of the regression is 3.1%, the F-statistic equals 18.85 and the constant *a* has a t-ratio of -5.45 reflecting the bias in the forecasts discussed above. The standard errors of the three forecasts are 0.08, 0.06 and 0.07 with t-ratios of -0.27 for *VIXf*, 6.54 for *GJRf* and -1.67 for *INTRAf*. These t-ratios must be interpreted cautiously because the standard errors may be misleading. The high t-ratio for *GJRf* compared with that for *INTRAf* is counterintuitive. The evidence for some incremental information is more intuitive and can be attributed to the mismatch between the one-day ahead forecast horizon and the one-month horizon that defines *VIX*.

5. Conclusions

Previous studies of low-frequency (daily or weekly) index returns and implied volatilities have produced conflicting conclusions about the informational efficiency of the S&P 100 options market. Our in-sample analysis of low-frequency data using ARCH models finds no evidence for incremental information in daily index returns beyond that provided by the *VIX* index of implied volatilities. This conclusion is in agreement with the recent evidence of Christensen and Prabhala (1998) and Fleming (1998). The extension here of the historic information set to include high-frequency (five-minute) returns shows, furthermore, that there appears to be only minor incremental information in high-frequency returns; this information is almost subsumed by implied volatilities.

Out-of-sample comparisons of volatility forecasts show that *VIX* provides more accurate forecasts than either low-frequency or high-frequency index returns, regardless of the definition of realised volatility and the horizon of the forecasts. A mixture of the *VIX* forecasts and forecasts from index returns shows that there is probably some incremental forecasting information in daily returns when forecasting one-day ahead. For the longer forecast horizon of 20 trading days, that closely matches the life of the hypothetical option that defines *VIX*, daily observations of *VIX* contain all the relevant forecasting information.

Our results for equity volatility confirm the conclusion of Andersen and Bollerslev (1998), obtained for foreign exchange volatility, that intraday returns provide much more accurate measures of realised volatility than daily returns as well as providing more accurate forecasts. Although high-frequency returns are highly informative about future volatility, we have found that implied volatilities were more informative throughout our sample period from 1987 to 1999.

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Table 1

Models for the S&P 100 index, daily returns from January 1987 to December 1992

Daily index returns r_t are modelled by the ARCH specification :

$$r_t = m + y_1 d_t + e_t \ ,$$

$$e_t = h_t^{\frac{1}{2}} z_t, \qquad z_t \sim \text{i.i.d.}(0,1),$$

$$h_t = \frac{a_0 + a_1 e_{t-1}^2 + a_2 s_{t-1} e_{t-1}^2 + y_2 d_{t-1}}{1 - bL} + \frac{gINTRA_{t-1}}{1 - b_I L} + \frac{dVIX_{t-1}^2}{1 - b_V L},$$

d_t is 1 for 19 October 1987 and otherwise is 0,

s_t is 1 if e_t is negative, otherwise s_t is zero,

INTRA is a sum of squared intraday returns,

VIX is a measure of implied volatility.

Parameter	Model (1)	(2)	(3)	(4)	(5)	(6)	(7)
$a_0 \times 10^6$	2.6891	44.797	0.7298	9.6881	12.5372	6.3594	0.3110
	(1.53)	(3.85)	(1.48)	(0.74)	(0.72)	(0.55)	(0.70)
a_1	0.0136		0.0085		0.0741		0.0029
	(1.53)		(1.56)		(1.34)		(0.47)
a_2	0.0280		-0.0078		-0.0485		-0.0029
	(1.19)		(-0.79)		(-1.20)		(-0.47)
b	0.9417	0.9793	0.9773	0.5209	-0.3039	0.5954	0.9695
	(33.67)	(118.7)	(130.8)	(1.53)	(-1.93)	(3.58)	(95.89)
$y_2 \times 10^3$	1.563	0.590	0.432	2.104	0.800	1.618	0.259
	(2.13)	(1.67)	(1.26)	(1.40)	(0.81)	(0.68)	(0.67)
g		0.6396	0.5661			0.3718	0.3742
		(3.64)	(3.32)			(1.86)	(1.86)
b_I		0.2523	0.2350			0.0360	0.0539
		(1.51)	(1.53)			(0.23)	(0.39)
d				0.4313	0.4101	0.3283	0.2816
				(6.31)	(6.06)	(4.47)	(3.27)
b_V				0.1509	0.1553	0.1943	0.1778
				(2.73)	(2.59)	(3.40)	(3.49)
Log-L	4833.66	4845.80	4848.35	4851.97	4858.48	4859.70	4860.79
Excess log-L		12.14	14.69	18.31	24.82	26.04	27.13
R^2	0.197	0.318	0.307	0.377	0.367	0.376	0.358

Notes - Parameters are estimated by maximising the quasi-log-likelihood function, defined by assuming conditional normal densities. Robust t-ratios, calculated using GAUSS, are shown in brackets. Excess log-likelihood is relative to Model (1). Estimates of m range from 0.00039 to 0.00047. y_1 is -0.237. The constraints $a_1, a_1 + a_2 \geq 0$ are applied.

Table 2

The relative accuracy of volatility forecasts from January 1993 to December 1999

Realised volatility $y_{T,N}$ is defined by either $\sum_{j=1}^{N} \left(r_{T+j} - m\right)^2$ or $\sum_{j=1}^{N} INTRA_{T+j}$. Forecasts $x_{T,N}$ made at time T are obtained from historic volatility estimates HV, the GJR model, the INTRA series or the VIX series. These forecasts are defined in Section 3.2 and make use of equations 4 (HV), 5 (GJR), 7 (VIX) and 8 (INTRA). The accuracy of forecasts is measured by the proportion of explained variability,

$$P = 1 - \frac{\sum_{T=s}^{n-N} \left(y_{T,N} - x_{T,N}\right)^2}{\sum_{T=s}^{n-N} \left(y_{T,N} - \bar{y}\right)^2},$$

which is a linear function of the mean square error of the forecasts. Values of P are calculated from $1769 - N$ forecasts.

A. Values of P for forecasts of sums of squared excess returns

Forecast	$N = 1$	$N = 5$	$N = 10$	$N = 20$
HV	0.037	0.089	0.112	0.128
GJR	0.106	0.085	0.016	0.013
INTRA	0.099	0.204	0.214	0.250
VIX	0.115	0.239	0.297	0.348

B. Values of P for forecasts of sums of the intraday volatility measure *INTRA*

Forecast	$N = 1$	$N = 5$	$N = 10$	$N = 20$
HV	0.167	0.243	0.255	0.255
GJR	0.375	0.289	0.169	0.181
INTRA	0.383	0.494	0.455	0.465
VIX	0.401	0.533	0.534	0.545

Table 3

Correlations and multiple correlations for regressions of realised volatility on volatility forecasts from January 1993 to December 1999

Realised volatility is defined by either $\sum_{j=1}^{N}\left(r_{T+j}-m\right)^2$ or $\sum_{j=1}^{N} INTRA_{T+j}$. Forecasts made at time T are obtained from historic volatility estimates HV, the GJR model, the INTRA series or the VIX series. These forecasts are defined in Section 3.2 and make use of equations 4 (HV), 5 (GJR), 7 (VIX) and 8 (INTRA). The table documents the squared correlation R^2 from regressions of realised volatility on one or more forecasts. Values of R^2 are calculated from $1769-N$ forecasts.

A. Values of R^2 for forecasts of sums of squared excess returns

Explanatory variables	$N=1$	$N=5$	$N=10$	$N=20$
HV	0.043	0.111	0.151	0.197
GJR	0.118	0.181	0.189	0.223
INTRA	0.099	0.212	0.238	0.285
GJR & INTRA	0.119	0.217	0.240	0.287
VIX	0.129	0.249	0.304	0.356
VIX & INTRA	0.129	0.250	0.304	0.356
VIX & GJR	0.139	0.253	0.304	0.356
VIX, GJR & INTRA	0.144	0.253	0.304	0.356

B. Values of R^2 for forecasts of sums of the intraday volatility measure *INTRA*

Explanatory variables	$N=1$	$N=5$	$N=10$	$N=20$
HV	0.185	0.282	0.309	0.335
GJR	0.423	0.449	0.395	0.403
INTRA	0.385	0.506	0.482	0.499
GJR & INTRA	0.443	0.525	0.490	0.504
VIX	0.445	0.567	0.559	0.569
VIX & INTRA	0.448	0.575	0.563	0.576
VIX & GJR	0.491	0.586	0.564	0.576
VIX, GJR & INTRA	0.495	0.586	0.565	0.577

Figure 1
In-sample comparison of conditional variance forecasts of squared returns (logarithmic scale)
2 January 1987 to 31 December 1992

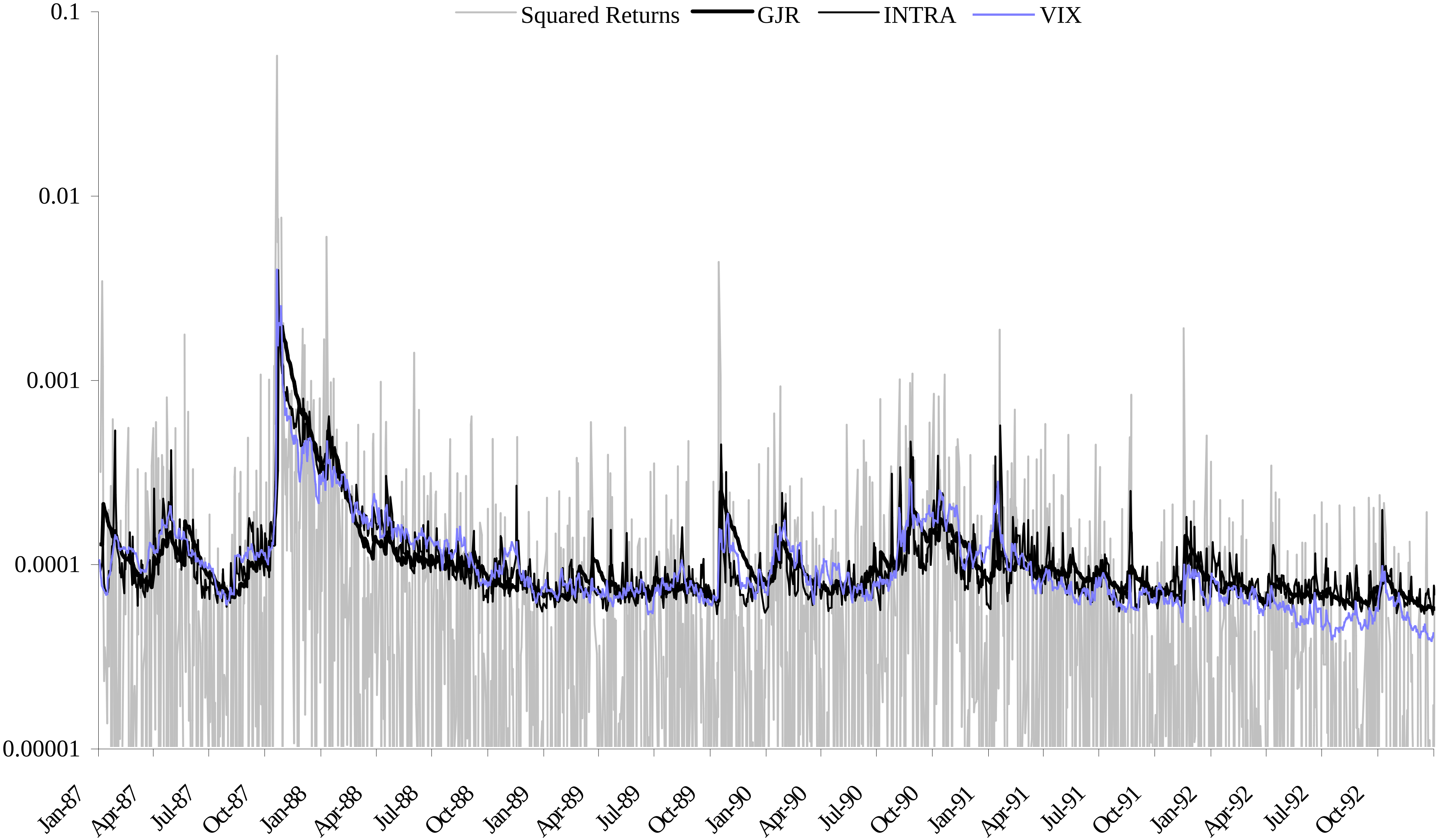

Figure 2(a)
Out-of-sample comparison of one-day ahead forecasts of squared returns (logarithmic scale)
4 January 1993 to 31 December 1999

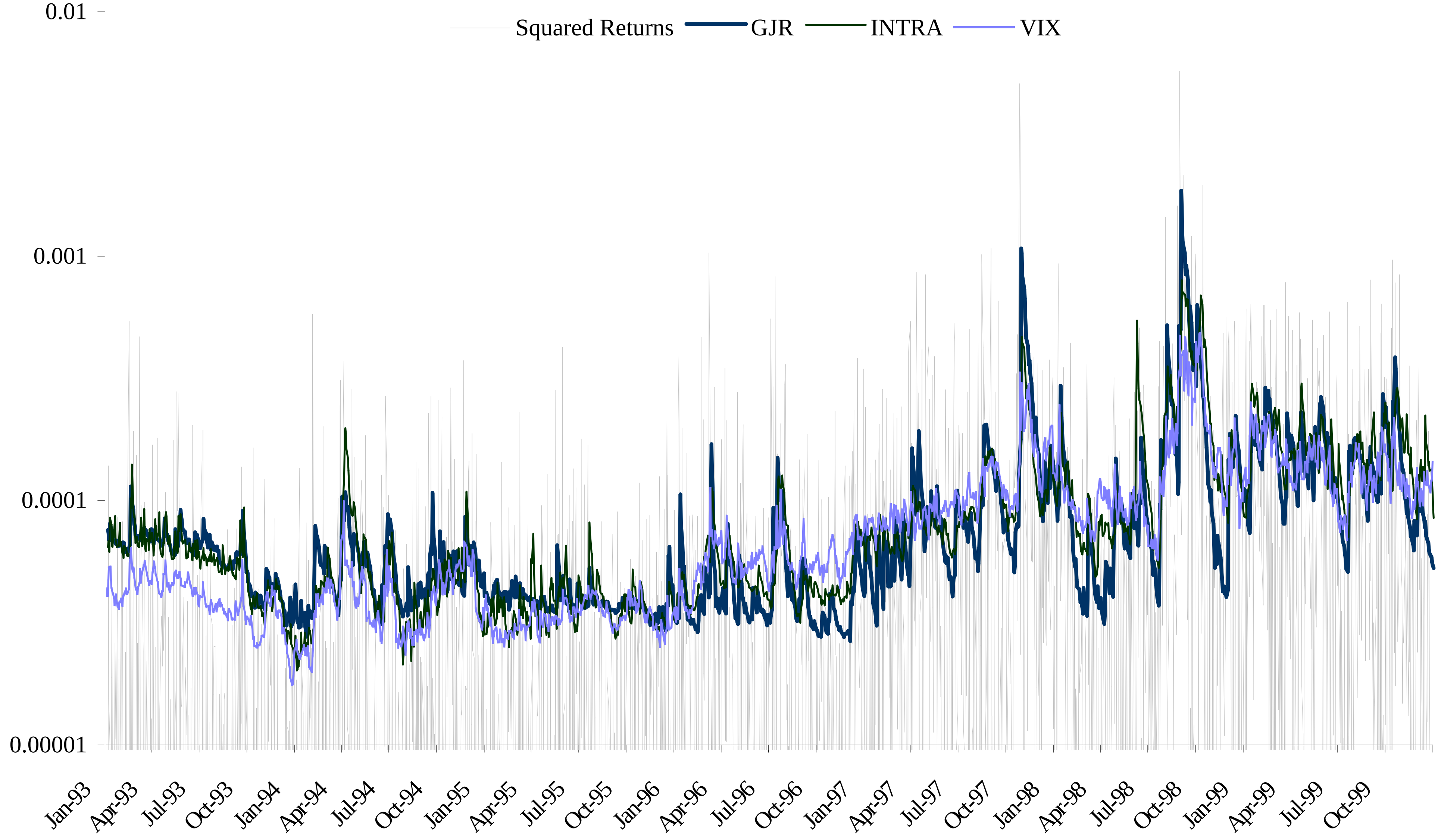

Figure 2(b)
Actual values and forecasts of realised volatility over 20 days, measured using intraday returns (logarithmic scale)
4 January 1993 to 31 December 1999

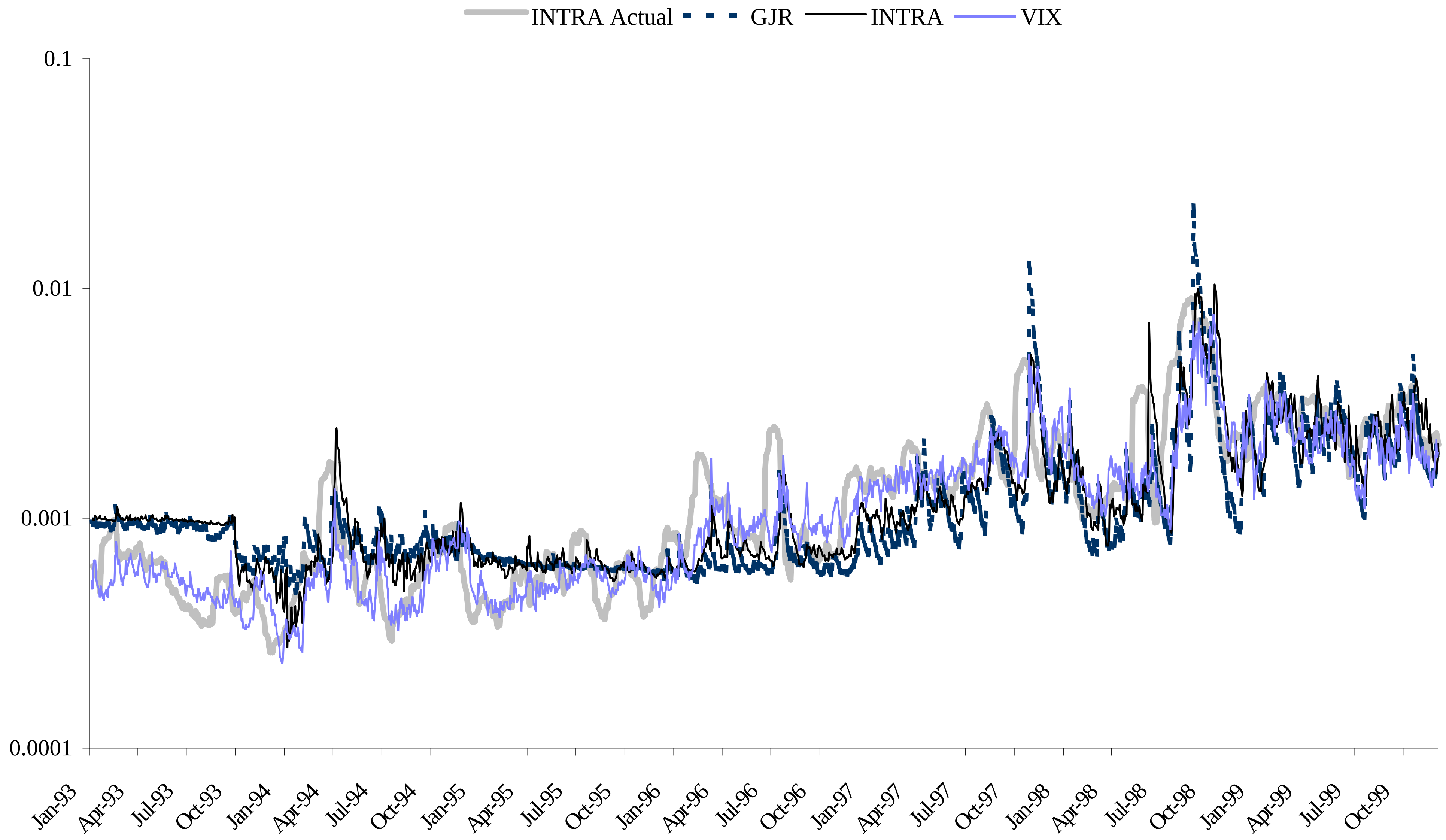

Figure 3
Scatter plot of actual values and VIX forecasts of realised volatility over 20 days, measured using intraday returns
4 January 1993 to 31 December 1999