A Probabilistic Approach to Worst Case Scenarios

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Historical simulation is a natural setting for scenario analy-
sis, but it must pay attention to current market conditions
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Value at Risk (VaR) is increasingly popular as a management and regulatory tool. To further its acceptance it is necessary to assess its reliability under conditions likely to be encountered in financial markets. A logical venue to investigate this issue is through the use of historical simulation.

Historical simulation relies on a uniform distribution to select innovations from the past. These innovations are applied to current asset prices to simulate their future evolution. Once a sufficient number of different paths has been explored it is possible to determine a portfolio VaR without making arbitrary assumptions on the distribution of portfolio returns. This is especially useful in the presence of abnormally large portfolio returns.

From the early days of modern finance large returns are known to cluster in time. The resulting fluctuations in daily volatility make the confidence levels of VaR computations that ignore clustering unreliable. This is the case with VaR measurements based on the variance-covariance matrix and Monte-Carlo methods, that typically ignore current market conditions to produce flat volatility forecasts for future days. Moreover the use of the covariance matrix of security returns or the choice of an arbitrary distribution in the Monte-Carlo method usually destroys valuable information about the distribution of portfolio returns.

To make our historical simulation consistent with the clustering of large returns we model the volatility of our portfolio as a GARCH process. Past daily portfolio returns are divided by the GARCH volatility estimated for the same date to obtain standardised residuals. A simulated portfolio return for tomorrow is obtained multiplying a randomly selected standardised residual by the GARCH forecast of tomorrow volatility. This simulated return is used to update the GARCH forecast for the following day, that is then multiplied by a newly selected standardised residual to simulate the return for the second day. Our recursive procedure is repeated until the VaR horizon (i.e.,10 days) is reached, generating a sample path of portfolio volatilities and returns. We repeat our procedure to obtain a batch of sample paths of portfolio returns. A confidence band for the corresponding portfolio values is built by taking the Kernel (empirical) frequency distribution of values at each time. The lower 1% area identifies the worst case over the next ten days.

To illustrate our procedure we constructed a hypothetical portfolio, diversified across all thirteen national equity markets of our data sample. To form our portfolio each equity market is weighted proportionally to its capitalisation in the world index as on December 1995. The portfolio weights are reported in the table below:

Table 1 : Portfolio weights

country	our portfolio	world index (dec 95)
Denmark	0.004854	0.004528
France	0.038444	0.035857
Germany	0.041905	0.039086
Hong Kong	0.018918	0.0176450
Italy	0.013626	0.012709
Japan	0.250371	0.233527
Netherlands	0.024552	0.022900
Singapore	0.007147	0.006667
Spain	0.010993	0.010254
Sweden	0.012406	0.011571
Switzerland	0.036343	0.033898
UK	0.103207	0.096264
US	0.437233	0.407818

Hence these weights are held constant for the entire 10 year period and multiplied by the thirteen local index returns. Since market risk needs to quantify eventual portfolio losses in one currency all local portfolio returns are measured in US dollars. The descriptive statistics together with the Jarque-Bera[1] normality test are reported on table 2. Figure 1 displays the empirical distribution of portfolio's returns.

Table 2 : Descriptive statistics of the equally weighted portfolio

mean (p.a.)	std. dev (p.a.)	skewness	kurtosis	normality	p value
10.92%	12.34%	-2.828	62.362	3474.39	0.000

The last column is the probability that our portfolio returns are generated from a normal distribution.

The rejection of normality in table 1 and the pattern of clustering visible in figure 1 led us to model our portfolio returns, r_t, as a GARCH process with asymmetries, with daily volatility h_t given by eq.1:

$$r_t = 100 \cdot \mathrm{Ln}\left(\frac{P_t}{P_{t-1}}\right) + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \qquad (1.a)$$

$$h_t = \omega + \alpha\left(\varepsilon_{t-1} + \gamma\right)^2 + \beta h_{t-1} \qquad (1.b)$$

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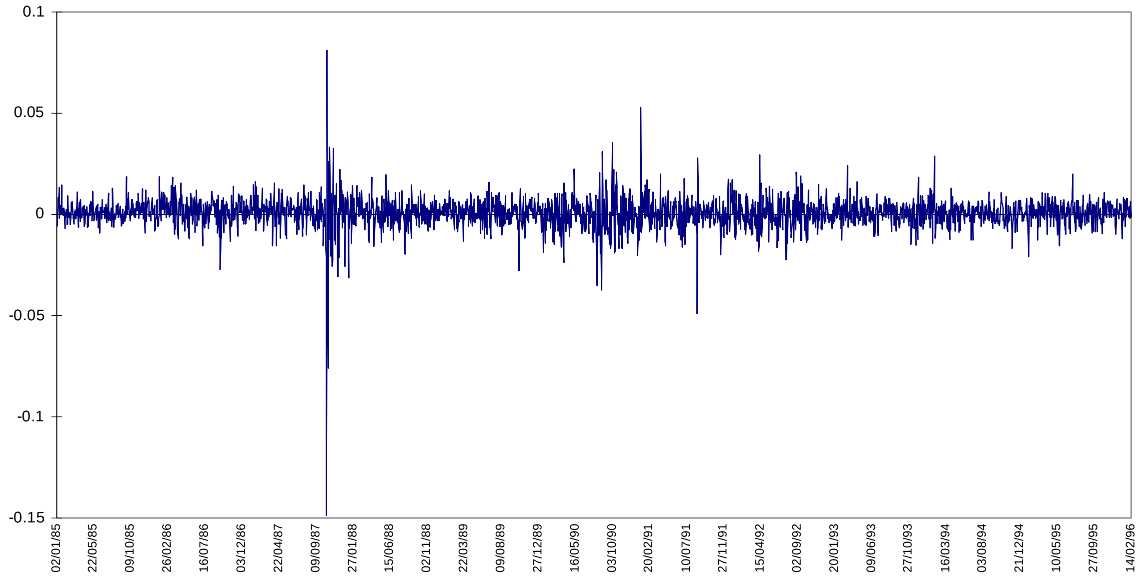
[1] The test for normality is the Jarque-Bera test,
$$JB = \left(\left(\text{skewness}^2 / 6\right) + \left(\text{Kurtosis} - 3\right)^2 / 24\right) \xrightarrow{\text{law}} \chi^2_{2\,df}$$

Fig 1 : World Capitalisation weigthed Portfolio Returns



Therefore our portfolio volatility is modelled to depend on the most recently observed portfolio return. The combination of GARCH volatility and portfolio historical returns offers us a fast and accurate measure of the past, current (and future) volatility of the current portfolio. No estimation of the correlation matrix of security returns is required. Furthermore our VaR method contains fewer "bad surprises", since GARCH models allow for fat tails on the unconditional distribution of the data. The effects of our choice become apparent if the returns in figure 1 are compared with figure 2, where the returns have been scaled by their daily volatility, as in equation 2:

$$z_t = \frac{r_t}{\sqrt{h_t}} \tag{2}$$

Clustering of returns in time is reduced by volatility scaling and the distribution of returns now appears to be more uniform. However the large number of returns still exceeding 3 standard deviations suggests that our scaling does not make returns normal. Our annualised portfolio volatility, in figure 3, varied from 7 to 21 over 10 years.

Fig 2 : Porfolio Stress Analysis
(Standardised Residuals)

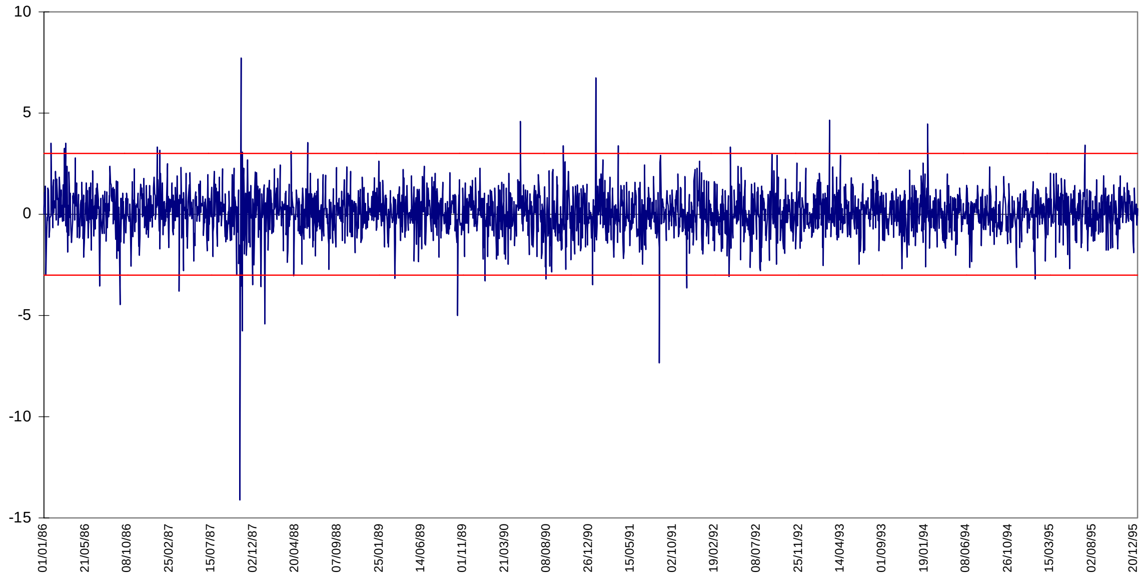
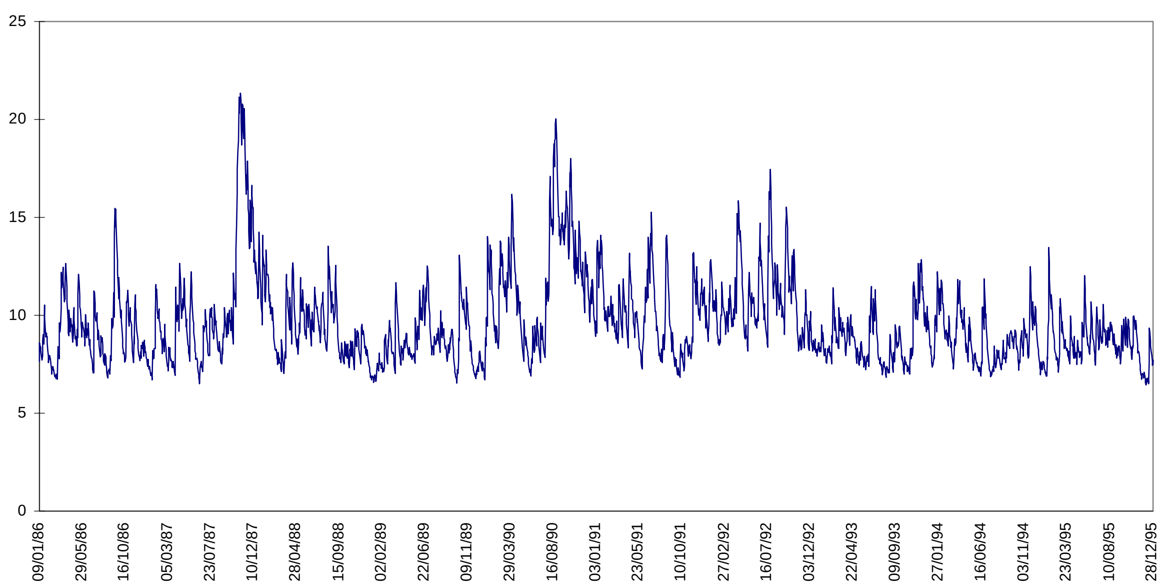


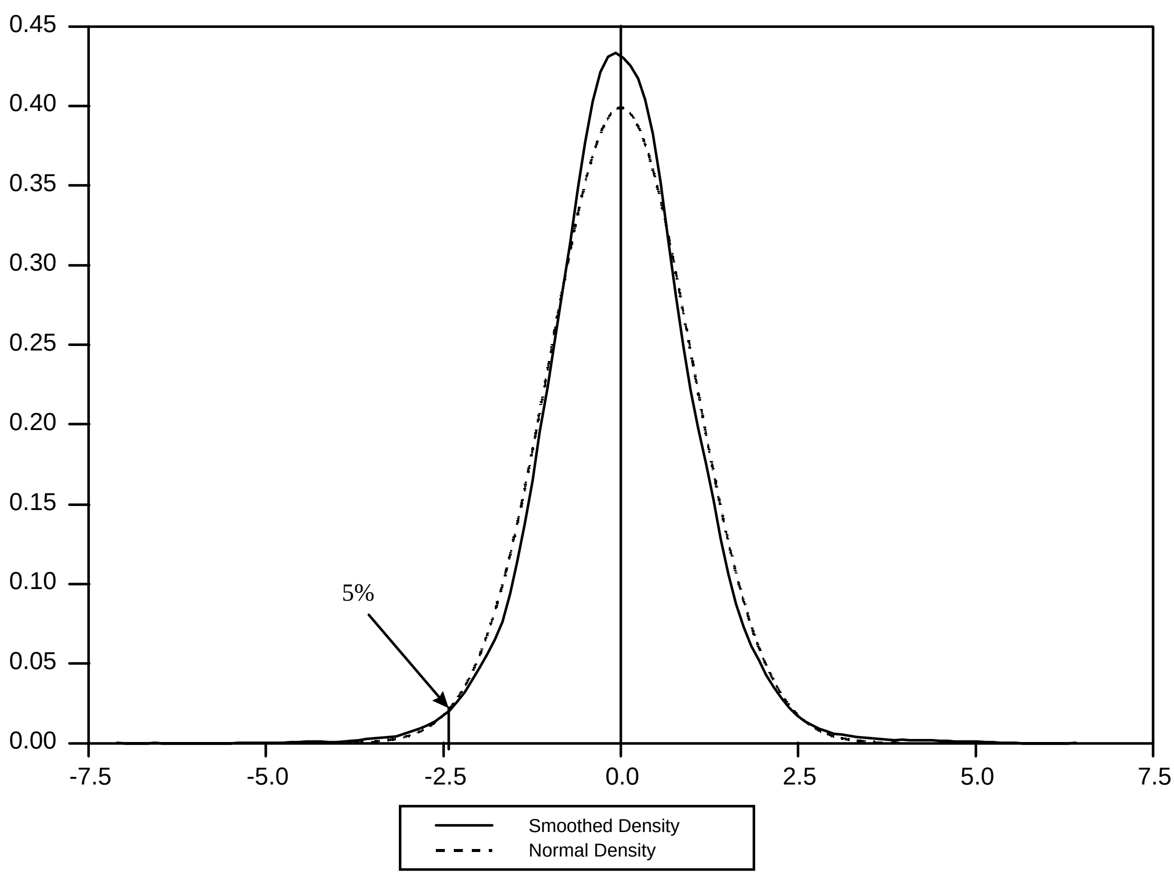
Fig 3 : Annualized volatility of the portfolio



The scaled returns are the foundation of our simulation. To simulate portfolio returns over next 10 days we select randomly 10 returns from figure 2. We then construct iteratively the daily portfolio volatility that these returns imply according to equation 1. We use this volatility to rescale our returns. The resulting returns reflect therefore current market conditions rather than market conditions associated with returns in figure 1.

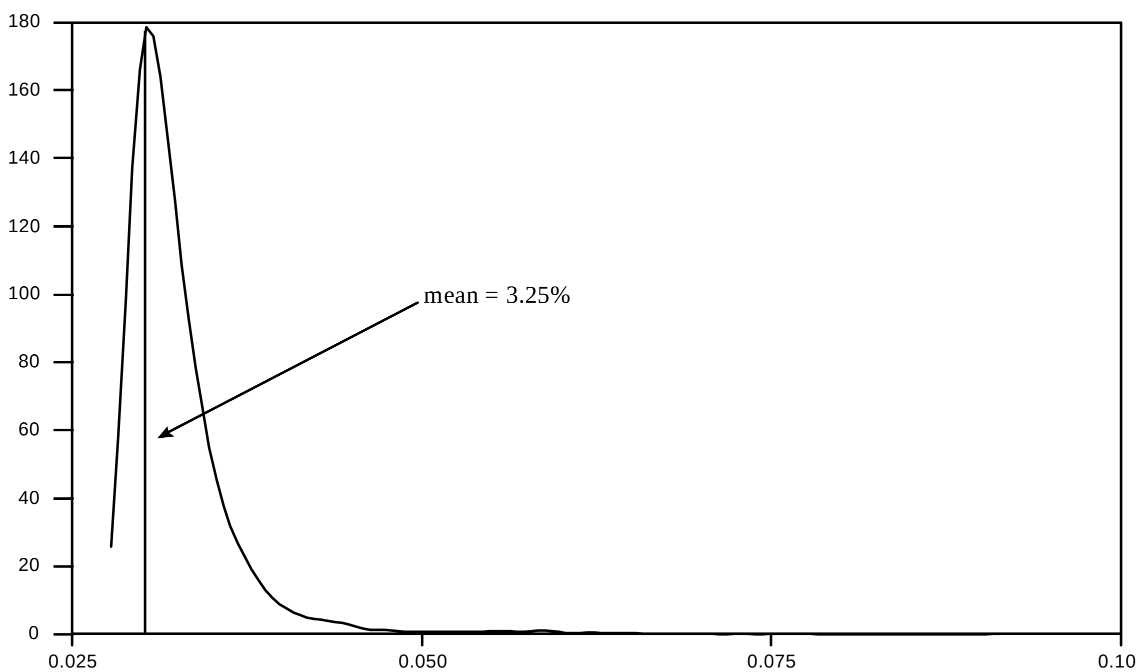
To obtain the distribution of our portfolio returns we replicated the above procedure 10,000 times. The resulting -normalised- distribution is shown in figure 4. The normal distribution is shown in dots in the same figure for ease of comparison.

Fig 4 : Normalized Estimated Distribution of Returns in 10 days versus the normal density (10,000 Simulations)



Not surprisingly, simulated returns on our well-diversified portfolio are almost normal, except for their steeper peaking around 0 and some clustering in the tails. The general shape of the distribution supports the validity of the usual measure of VaR for our portfolio. However a closer examination of our simulation results shows how even our well-diversified portfolio may depart from normality under worst case scenarios. There are in fact several occurrences of very large negative returns, reaching a maximum loss of 9.52 . Our empirical distribution implies losses of 3.38 and 2.24 at confidence levels of 1 and 5 respectively.

Fig 5 : Estimated Distribution of Portfolio VaR in 10 days (10,000 Simulations)



The reason for this departure is the changing portfolio volatility and thus portfolio VaR, shown in figure 5. Portfolio VaR over next 10 days depends on the random returns selected in each simulation run. Its pattern is skewed to the right, showing how large returns tend to cluster in time. These clusters provide realistic worst case scenarios consistent with historical experience. Of course our methodology may produce more extreme departures from normality for less diversified portfolios.

In conclusion, our simulation methodology allows for a fast evaluation of VaR and worst case scenarios for large portfolios. It takes into account current market conditions and does not rely on the knowledge of the correlation matrix of security returns.

Our computations were performed using the RiskClock software.
A full description of it is available from the authors.