COMPARING DIFFERENT METHODS FOR ESTIMATING VALUE-AT-RISK (VaR) FOR ACTUAL NON-LINEAR PORTFOLIOS: EMPIRICAL EVIDENCE*

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Abstract

The purpose of this paper is to compare the different estimation methods of Value-at-Risk (VaR) as a market risk measurement of actual bank non-linear portfolios (specifically comprised of currency options) in the context of the supervision of bank solvency. The aim is to establish the best method given these specific circumstances.

The main conclusion is that, when estimating VaR for non-linear actual portfolios, in a context of supervision of bank solvency, the precision of the Monte Carlo simulation method is to be preferred to the speed that can be obtained with the variance-covariance matrix analytic method.

We obtain for it, theoretical evidence as well as an empirical one. The originality of the empirical study consists in using for it a non-linear actual portfolio: the currency options portfolio of one of the biggest Spanish banks, and dated the 21st October 1996.

Keywords: Value-at-Risk (VaR), Estimation Methods, Monte Carlo Simulation, Variance-Covariance Matrix Method, Currency Options, Non-Linear Portfolios, Supervision, Bank Solvency, Empirical Evidence.

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1. INTRODUCTION

The purpose of this paper is to compare the different estimation methods of Value-at-Risk (VaR) as a market risk measurement of non-linear actual banking portfolios (specifically comprised of currency options) in the context of the supervision of bank solvency. The aim is to establish the best method given these specific circumstances.

The VaR of an asset or a portfolio is the maximum expected loss (measured in currency units over a given time period and at a given level of confidence, under normal market conditions.

Conceptually, following the established definition, the VaR calculation simply means determining a percentile and the different VaR estimation methodologies are, in fact, the classic statistical techniques of percentiles estimation, which at present, are undergoing a “renaissance” in the context of the VaR[1].

However, although VaR is conceptually simple, its implementation in practice by the banks is not so straightforward, since there are several alternative methodologies; each with their advantages and disadvantages, derived from the underlying hypotheses of such methodologies.

At present, the three[2] existing methodologies for measuring VaR are the analytic method of the variance-covariance matrix, the historic simulation approach and the Monte Carlo simulation, although within each methodology, the variants are many.

Depending on the realistic, restrictive or simplifying nature of the underlying hypotheses, the results of the application of the methodology, that is, the VaR estimate obtained from applying such a methodology, will be more or less reliable and precise.

In the present-day context of the increasing globalisation of markets and a greater internationalisation of economies, the understand of the impact that the changes in market conditions have on the solvency of banks, is becoming more and more important,. The above-mentioned impact can be measured through the VaR. It is important for the banks themselves as well as for the regulators and supervisors, who try to avoid the contagion of the individual banking crises to the whole financial system, thereby avoiding so-called systemic risk.

Others factors which have favoured the appearance of new market risk measures (among them the VaR) are financial innovations and the spectacular growth of derivative products in terms of volume of trading as well as in terms of complexity.

The misuse of derivative products seems to have contributed, on occasion, to the enormous losses experienced by prestigious financial institutions. This has provoked real apprehension, concern and confusion over the use of derivative instruments and has generated an extensive debate about the risks resulting from the use of these instruments. Up to the point of identifying them as responsible for the most recent

[1] Additional progress the field of the computer science or information technology is allowing for the recover of such techniques for use in real time, as in the case of the Monte Carlo Simulation.

[2] For some authors, a fourth methodology could be the application of the Extreme Value Theory. However, in our opinion, such theory is useful as a complementary analysis of the VaR, rather than as a VaR estimation approach by itself. See also Coronado (2000a).

financial disasters such as *Baring, Daiwa, Sumitomo, Metallgesellschaft, Orange County, Long Term Capital Management (LTCM)*, etc. It is paradoxical that instruments such as derivatives originally conceived as a hedging instrument could be blamed for the generation of bigger risks, and more difficult to control[3].

A common element in all these crises was the existence of deficient systems of measurement, management and risk control.

Not only have derivatives been decisive in the appearance of the VaR, but specifically the most important challenge for current risk management consists of the VaR calculation of derivative instruments, particularly options, which are the subject of our research.

After noting present-day importance, the reason for which we felt impelled to carry out this research has been a real need and relevance, as two facts proved it:

1. Since January 1998, the countries of the Group of Ten (from which the Basle Committee on Banking Supervision was created) and since January 1996, the Member States of the European Union are obliged to demand from their banks the measurement of their market risk using the VaR. They can choose among several methods: either the legislated standard model or any of the internal models created for that purpose by the banks themselves; providing that they fulfil a list of requirements in order to count on the prior approval of the supervisors. The capital requirements to face the market risk are established according this VaR measurement.

2. The choice of the methodology that was used for the VaR estimation gives rise to very different results of the VaR[4].

Bearing in mind the two above mentioned facts, we can deduce that a same bank with the same risk should have a different capital depending only on the chosen method for measuring VaR. This means a drawback for the bank (if the requirements are higher than those which are really needed) as well as for a precise and reliable estimation of the bank's solvency, by the supervisors. Instead of securing the stability and solvency of the financial system, the opposite effect may even happen, since such a situation would end up with a tendency for the banks to use those VaR methods which produce lower levels of VaR and not the one which best measures the risk in each specific case.

Therefore, this research should be seen as current, necessary and important.

In fact, in order to measure reliably the degree of solvency of the bank, but at the same time, without requiring more capital than necessary, one should research which of the possible methods is best.

It can be deduced then the importance of establishing the best method of measurement of the VaR that is acceptable to the supervisor in a manner that does not

[3] See Coronado (2000b) for a more detailed study of this debate.
[4] Among the works which highlight large variations in the VaR estimates depending on the methodology used, we refer to: Gizycki and Hereford (1998), Pritsker (1997), Jackson et al. (1997); Fallon (1996), Hendricks (1996), Marshall and Siegel (1996), Beder (1995 a and 1996); Estrella et al. (1994).

impose “extra” capital requirements and at the same time allows the bank to manage its risk in a responsible and prudent fashion, and consequently its solvency.

And the first conclusion we arrive at in previous study, is that a best VaR estimation method, in general or absolute terms, does not exist, but it depends on two factors, which always need to be kept in mind in mind in order to secure a set of conclusions:

1. The kind of portfolio for which we wish to calculate the VaR, and
2. The use we wish to give to this VaR estimation.

Therefore, we make the comparison for a specific kind of portfolio: non-linear actual portfolios (in short actual portfolios of currency options of one of the biggest Spanish banks); and in a specific context of use of the VaR estimate: in the context of the supervision of bank solvency.

The decision about the kind of portfolio that we use in our comparative survey has not been accidental nor irrelevant: we have chosen an actual option portfolio (that is to say, non-linear actual portfolios), for two reasons:

1. **No study has previously done so.** As we will show in part 4, the empirical studies that compare different approaches to estimating VaR, have no done it for portfolios containing options, nor do they do it with actual portfolios. In addition, very few specify the context in which they develop their comparative study (that is to say, the use given to these VaR estimates).

2. **It is the most complicated case**[5], as it can be proved theoretically (and, in fact, we do it) and also empirically. That is the conclusion, for instance, of the GYZYCKI and HEREFORD survey (1998); which is the only one of all the existent empirical surveys that proposed to send a series of portfolios of different kinds (among them some non-linear) to all the Australian banks, to have the daily VaR of each portfolio calculated with their own methods, and thus analyse the dispersion in the results. Of all the banks which answered (only twenty-two), just two banks were able to calculate the VaR for all the portfolios. Those that contained options (or any kind of non-linearity) presented more problems to the banks who were unable to calculate the VaR for them. Of course, the number of banks that used the Monte Carlo simulation method to calculate the VaR was even smaller. Regarding the Spanish banks in that sense, we also saw “how the land lies”, and the conclusion was the same.

We would like to emphasize the fact that we carried out the empirical research with an **actual portfolio** of one of the big Spanish banks; which in addition to constituting a novel or original contribution with respect to those studies that had been carried out previously, also increased the difficulty of the research. It has been extremely difficult to be able to obtain actual portfolios of the Spanish banks. Not even a promise of

[5] Estimating the VaR for options, is the most complicated case. But also, our survey, analyses currency options, where the fact of the fat tails is particularly significant, increasing the difficulty. We are going to talk about it later.

confidentiality convinced the persons in charge of the banks' departments of risk; even less when the banks find themselves in a full phase of implementing this new risk measurement, VaR, given its novelty..

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2. VaR: THEORETICAL FOUNDATIONS

2.1.- DEFINITION

The VaR can be defined as:

> The maximum expected loss (measured in currency units) in an asset's value (or a portfolio) over a given time period and at a given level of confidence (or with a given level of probability), under normal trading conditions.

The VaR provides the answer to the following question: What is the largest amount I can lose with a probability of *x%* for a period *y*?

For example, a portfolio manager could state that the monthly VaR amounts to 20 million pesetas with probability of 99%; that means that for next month, there is only a 1% probability that the loss would be higher than 20 million pesetas.

Thus, we can define formally the VaR as follows:

Supposing that:

$V(P_t, X_t, t)$: is the value at the moment t of the portfolio V, comprised of the instruments X_t whose prices are P_t.

$\Delta V(P_{t+1} - P_t, X_t, t)$: is the change or variation of the portfolio value V, in the period between t and t + 1.

The cumulative probability function of $\Delta V(P_{t+1} - P_t, X_t, t)$ conditioned by X_t and with the information at the moment t, I_t, is:

$$G(k, I_t, X_t) = P(DV(P_{t+1} - P_t, X_t)£\, k\, ô I_t) \qquad (1)$$

And its inverse is:

$$G^{-1}(a, I_t, X_t) = \inf\{k : G(k, I_t, X_t) = a\} \qquad (2)$$

For a confidence level α, the VaR can be defined in terms of the inverted cumulative probability function of ΔV:

$$VaR(a, I_t, X_t) = G^{-1}(a, I_t, X_t) \qquad (3)$$

The definition given by the formula (3) is equivalent to defining the VaR for a confidence level α as the percentile α of the distribution of $\Delta V(P_{t+1} - P_t , X_t)$given I_t..

Formula (3) shows how the VaR definition depends on the function G^{-1}, which is conditional on the instruments X_t and the information I_t . Therefore, it depends on the combined distribution of the instruments, which can become very complex when the number of different instruments is big.

If we call *f* the vector of risk factors or underlying independent variables, which depend upon the price of the instruments, (we can call *n* the number of factors) and ε_{t+1} the vector nx1 of the variations or changes in such risk factors, for the period between t and t+1 ($\varepsilon_{t+1} = f_{t+1} - f_t$), we can express the changes in the value of the Portfolio V, for the period between t and t+1 as $\Delta V(e_{t+1}, X_t, t)$.

Thus, within this statistical framework, we focus specially on the study of two issues, which are key ones in order to calculate VaR in general, and the VaR of the options in particular. The validity of the **normality assumption** for the value or return on an asset or portfolio (including the study of **fat tails** as well as the **volatility clustering**). And **the assumption of non-linearity** of the assets, for which we want to estimate the VaR.

2.2.- THE VAR FOR PARAMETRIC PROBABILITY DISTRIBUTIONS: VALIDITY OF THE NORMALITY ASSUMPTION IN AN ASSET RETURNS DISTRIBUTION: RECENT CONTRIBUTIONS

There are two ways to obtain the VaR from the probability distributions or return[6] of an asset or portfolio:

- Through the so-called **general approach**, that is to say, having in mind the "real" empirical distribution of the portfolio or asset returns. In this case, we get the VaR from its corresponding percentiles. It includes the VaR estimation methods of historical simulation and Monte Carlo simulation.
- Through the **parametric approach**, in which a parametric distribution is used which is adjustable as much as possible to the data. It includes the analytic method of the variance-covariance matrix in order to calculate the VaR

[6] In practice, people work with returns instead of prices, because the former have more attractive statistical properties (see Longerstaey et al. (1996), p. 46. Besides, out of the possible returns (price changes) that can be defined, the usual way to work is with the continually composed return (or the difference of the prices' logarithms). Almost all the authors whom we analysed do it in this way.

The **key aspect** in the parametric approach is **the assumption made on the returns PDF (probability density function)**. In practice, the easiest and more frequent assumption is that this function corresponds to a normal distribution. Such an assumption of normality has the advantage of simplifying the VaR estimation; since the VaR can be derived directly from the standard deviation of the portfolio returns using a multiplying factor that will depend on the confidence level chosen. Moreover, it facilitates the comparison of VaR values calculated with different confidence levels and time periods.

The most significant advantage of the general approach in comparison with the parametric one is that it does not require the establishment of any previous assumption concerning the volatilities of the returns, nor about the correlations between them, nor about the nature of the distributions themselves. In particular, it does not require portfolio returns to have normal distributions.

2.2.1.- The Fat Tails problem

Since the VaR is based on the tails behaviour of the financial returns, specifically on the left handside, it is obvious that the issue of fat tails is basic to our study.

There has been a significant amount of empirical research on this topic in the last 25 years. However it is only recently, since the introduction of VaR as a market risk measure, that very important recent contributions are being made, not only at a theoretical level, but also in terms of empirical and with an open field of research for the future.

Therefore, in addition to the traditional empirical studies which confirmed the proven aspects of the pioneering studies by MANDELBROT (1963) and FAMA (1965), about the non-normal features of the financial returns (fat tails and excess kurtosis; asymmetry, *volatility clustering*), we have conducted an analysis and follow-up on the most recent contributions.

Among the empirical studies that have proven that the returns have the before mentioned characteristics and not exactly those of a normal distribution, it is necessary to refer to:

PRAETZ (1972), BLATTBURG and GONEDES (1974), KON (1984), JORION (1988), JANSEN and de VRIES (1991), TUCKER (1992), GLOSTEN et al. (1993), KIM and KON (1994); **LOGIN (1996), LONGIN (1997 a, b), DANIELSSON and de VRIES (1997 b), KLÜPELBERG et al. (1998), McNEIL (1998) and HUISMAN et al. (1998)** for stock prices. ROGALSKI and VINSO (1978), KOEDIJDK et al. (1992), **WILSON (1993),** de VRIES (1994), JORION (1995c), **ZANGARI (1996 a), ZANGARI (1996d), HUISMAN et al. (1997),** ALEXANDER and WILLIAMS (1997), **VENKATARAMAN (1997),** DANIELSSON and de VRIES (1997C), CORREDOR CASADO et al. (1998), LIM et al. (1998) and **HULL and WHITE (1998)** for exchange rates[7], and BOLLERSEY (1986), BOLLERSEY (1987), GHOSE and KRONER (1993), RUIZ (1994), HARVEY et al. (1994), **LONGERSTAEY et al. (1996),** HARVEY AND SHEPARD (1996), DUAN (1997), CAMPBELL et al. (1997), KEARNS and PAGAN (1997) and **LUCAS (1997 b)** for all markets.

[7] The problem of the fat tails is especially important and characteristic, especially in connection with currencies. See Jorion (1995) or Venkataraman (1997), p. 3.

In boldface, we highlight those surveys that deserve special attention, due to their current importance and interest and their direct application to VaR. We must emphasise that they are **the most recent studies on the subject of fat tails and VaR estimation**.

Thus, in such studies, it is identified **how estimates of VaR based on an assumed normal distribution of portfolio returns could underestimate (and overestimate) "true" VaR, and therefore, the capital required to cover the losses derived from market risk, endangering the actual solvency of the bank**.

This has led to many authors to propose (in order to describe the behaviour of returns) **other distributions with fatter tails than the normal distribution** (for example, Pareto's stable distribution, student's t distribution, the normal mixture approach, or the generalised error distribution); thereby allowing, for the modelling of the biggest movements in order to avoid erroneous VaR estimates. Even the most recent studies propose distributions, not to describe the returns behaviour (of all of them), but to outline only the behaviour of the extreme returns, that is to say, **those based on the Extreme Value Theory**

2.2.2.- The normality assumption in the case of options

The normality of the underlying asset does not mean that the option should follow a normal distribution, since the price of the options does not change in a linear fashion with reference to the underlying ones. Nevertheless, the normal distribution can be used, with sufficiently large portfolios of independent options, (by applying Central Limit Theorem).

Therefore, in order to determine when the options portfolio returns can be described by a normal distribution, we have to identify two aspects: first, the number of options in the portfolio (*i.e.* its size). Second, analyse if even though the options are not completely independent but with a small degree of serial correlation, a normal portfolio can be obtained also if the magnitude of the correlation is not large enough; thus determining the degree of self-correlation allowable.

The conclusions[8] reached are that only in the case of a portfolio with at least 20 independent options, the distribution would be close to normal; and if the options of the portfolio were not absolutely independent, the portfolio distribution would not be normal, however large the size. Not even for small degrees of self-correlation (for example, 10%) can the normality of the portfolio be achieved by increasing its size. Moreover, the non-linearity circumstance of the options causes, in turn, non-normality.

2.3.- NON-LINEAR MODELS OF FINANCIAL ASSETS VALUATION AND ITS LINEAR TAYLOR SERIES APPROXIMATION: THE PARTICULAR CASE OF OPTIONS.

As we have just seen, to calculate the VaR, the risk manager needs to estimate either the probability distribution of the future value of the asset or the portfolio and this estimated distribution must be similar to the "actual" or empirical distribution, as much

[8] See Finger (1997).

as possible. That means the risk manager must create a financial model which could describe such value. And this is, truly, the fundamental problem for obtaining the VaR.

This can be seen without applying a non-linear model of options valuation. It can be done with a simple quadratic approximation corresponding to the value of such an option[9].

Taking the variance of both expressions, we get:

$$Variance\ (dC) = D^2\ Variance\ (dS) + \left(\frac{1}{2}\ G\right)\ Variance\ (dS^2)$$
$$+\ 2\left(D\frac{1}{2}\ G\right)\ Covariance\ (dS,\ dS^2)$$

If the dS variable is distributed normally, the last term of the equation disappears and besides it is stated that: Variance (dS^2) = 2 Variance $(dS)^2$:

Thus:

$$Varianza\ (dC) = D^2\ Varianza\ (dS) + \frac{1}{2}\left[\Gamma\ Varianza\,(dS)\right]^2$$

and the approximate VaR[10] of the option is:

$$VaR\ (dC) = c\sqrt{\Delta^2 S^2 s^2 + \frac{1}{2}\left[\Gamma S^2 s^2\right]^2}$$

This proves the non-linear relationship between the VaR of the option and the VaR of the underlying asset.

Due to the complexity of estimating the VaR for non-linear positions, some authors choose to value such positions in a partial way, through the Taylor Theorem (delta or delta-gamma valuation).

We must point out that the fact of increasing the linear approximation (delta) to the quadratic one (gamma) presents very important **limitations**. ESTRELLA (1996) analyses correctly this drawback and warns about the necessity to apply Taylor approximations in the case of the options valuation with some caution. After checking

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[9] Jorion (1997), p. 144.

[10] As we have started from a value, also approximate, of the option.

the extended use[11] of the delta or the delta-gamma approximation of the value of an option, Estrella shows how a Taylor approximation, applied to the Black-Scholes options valuation model in terms of the prices regarding the underlying stock, changes as function of such prices. On the other hand, if the approximation is used in terms of the logarithmic prices, no divergence will occur.

His **two main conclusions** are:

1. Taylor series approximation should not be used, especially for stress testing.
2. Citing him literally (p. 375):
   "In risk management applications involving a preponderance of relatively small moves, it may be feasible -though sometimes risky- to use Taylor approximations. For moves no larger than one standard deviation, the accuracy of gamma approximations seems generally adequate. Problems may arise however, if attention is focused on the tails of the distribution as is often the case in risk management applications. (…) ***Special care should be used when approximating the values of highly nonlinear options, such as near-the-money short maturity options".***

For his part, SCHACHTER (1995) refutes each one of these conclusions, although without too much force, according to us. Besides, another drawback is that, when the gamma is incorporated, it is admitted the non-linear function of the variations in the option value with regard to the underlying. Therefore, **the normality assumption cannot be included in his distribution**. In fact, the delta-gamma approach approximates the value changes through a sum of lineal combinations of normally distributed variables and second order terms, which have a chi-square distribution. As a result, the normality is lost..

On the other hand, the gamma does not produce better approximations, in all the cases; especially in situations close to the option expiration date, or when the price of the underlying one is near to the price of the option.

If such linear approximation is exact enough or not, will depend on the portfolio composition and on the use that the trader gives to the measure of the resultant risk.

All things considered, the problem is what to do when returns are non-linear functions of the risk factors (as is the case with options) or when the risk variables themselves are non-normal. This is precisely the point of our study.

3. DIFFERENT METHODS TO MEASURING VaR: THEORETICAL COMPARATION

As MINNICH says (1998), p.41, *"One of the most difficult aspects of calculating VaR is selecting among the many types of VaR methodologies and their associated assumptions."*

[11] As Estrella himself remarks, it is even recommended as the standard model in the Capital Adequacy Directive (1993) or in the Basle Committee on Banking Supervision proposal (1995). Besides, a survey of the Group of Thirty (1994) test that the 98% out of the 125 operators who answered, used delta and 91% used gamma.

On Table 1, we have classified current different VaR estimation methods, with all their possible variants. The nomenclature used to describe every method differs significantly depending on the author, and it also contributes to confusion at times.

We have compiled different author's nomenclatures, in addition to the most accepted one.

It is not our goal to describe fully all the methods, but to assess instead the hypothesis sustaining them, therefore emphasising the underlying pros and cons (both coming from those underlying hypothesis) in order to make a theoretical comparison.

1. Analytic Method of the Variance-Covariance Matrix:

Both advantages and drawbacks of the variance-covariance matrix analytic method are consequences of the **underlying assumptions** in which it is based. Basically, the assumption about the joint normal distribution of the portfolio returns and the assumption about the lineal relationship (or quadratic at the most) among market risk factors (or independent variables) and the portfolio value.

The **advantages** of assuming those two hypotheses for the VaR estimation can be summarised in two:

1. It **facilitates** VaR **calculations**, making this method easily understandable for all the connected people in the risk management. Its implementation could be effortless.
2. **Rapidity** in such calculations, a very important feature when working in real time.

The principal **drawbacks** of this method are three:

1. The VaR estimation through the variance-covariance matrix approach gives VaR overestimates for small confidence levels and VaR underestimates for big levels of probability. As a result, it causes excesses or lacks in the required bank capital in order to face the market risk by the supervisor authorities, with the resulting repercussions in the bank solvency. This drawback comes from assuming normality in the portfolio returns.
2. The linearity assumption makes this method **only applicable, theoretically, to linear portfolios;** not a very useful feature, considering the great and growing use of non-linear assets (especially options) in banks portfolios.
3. Even by increasing the portfolio value approximation to a quadratic one (delta-gamma methods), success would not be guaranteed since an accurate VaR estimate of the non-linear portfolios is not feasible. We should also bear in mind, moreover, that such an increase means reducing the simplicity of this method (which was, as a matter of fact, one of its advantages), because of the additional assumptions that it needs (as a result of the normality loss, when a second order Taylor approximation is applied).

These two last drawbacks are particularly pertinent regarding options near the money and close to expiration.

Before evaluating another method, it is worth mentioning that the famous RikMetrics^{TM} of J. P. Morgan is an analytic variance-covariance matrix method.

2. Historical Simulation Method:

The historical simulation approach is an easy method both to understand and to explain. It is also quite easy to implement. It is a non-parametric method, which does not depend on any assumption about the probability distribution of the underlying asset. Therefore, it allows capturing fat tails (and other non-normal characteristics), while eliminating the need for estimating and working with volatilities and correlations. It also avoids greatly the modelisation risk. It is a global valuation method, and consequently, excludes the necessity to establish approximations (such as those based on Taylor), which produce inaccuracy in calculations. Thus, it can be applied to all kinds of instruments, both linear and non-linear.

All those advantages give it a theoretical superiority as opposed to the variance-covariance matrix Method, especially in the relevant aspect for us, which is the estimation of VaR for non-linear portfolios (particularly, currency options portfolios). Besides, this general theoretical superiority is supported by huge empirical evidence, as we have analysed in the previous epigraph. This method allows capturing fat tails; this is not the case of the variance-covariance matrix approach.

Nevertheless, the historic simulation method has got its own drawbacks too, related specially with the characteristics of the historic database used. This method depends completely on the specific historical data set used, and ignores any event not represented in such database. This is its main drawback: to either assume that future and past (as captured in the historical data set) will be alike; or to estimate the VaR from just one trajectory or chosen prices path.

This drawback, as we will see, is not present in the Monte Carlo simulation method, (although this last one has other disadvantages, of course).

There are two specific issues in this method that need to be cleared up. First to what extent does the reliability of this method deteriorate with the increase of the confidence level used. And second, the relationship of such reliability with the length of the data period used. We have to point out that, in neither case, the empirical evidence is conclusive.

3. Monte Carlo Simulation Method:

The Monte Carlo Simulation Method, is a global valuation one, parametric as well as non-parametric. Therefore, it excludes the necessity of establishing approximations (as those used in the variance-covariance matrix method), which introduce inaccuracy in VaR estimates. Thus, it can be applied to all kinds of positions, both linear and non-linear.

Besides, the non-parametric Monte Carlo simulation method, as it does not rely on any assumption about the probability distribution of the underlying asset, greatly avoids the modelisation risk and allows capturing fat tails (and other non-normal properties). At the same time, it excludes the necessity of estimating and working with volatilities and correlations, keeping out historical simulation method drawbacks as opposed to the variance-covariance matrix one.

All these advantages give a theoretical superiority to the non-parametric Monte Carlo Simulation method, as opposed to the variance-covariance matrix one, especially when estimating VaR for non-linear portfolios.

In addition to this, the parametric Monte Carlo simulation method presents a theoretical superiority as opposed to the variance-covariance matrix method. Although the former does call for the specification of a particular stochastic process for risk factors (dealing, therefore, with modelisation risk), it can be applied to non-linear positions and, it does not require the normality assumption.

And this happens not only as opposed to the variance-covariance matrix approach, **but also in contrast to the historical simulation method.** Its advantage lies in the random character of future prices paths, whereas prices generated by historical simulation represent only one of the possible paths that may happen.

Monte Carlo simulation offers a more realistic description of risk, since the distribution of price changes shows the full range of all the realisations and their probabilities.

In spite of being the most complex method to understand, to explain and to implement among the three approaches studied for estimating VaR, and in spite of being also the slowest one, **the Monte Carlo simulation method is the most powerful, flexible and accurate one for estimating VaR**.

However, its biggest drawback (the slowness) is not really so important, due to computer science development. This inconvenience will grow less important, as new techniques are being implemeted recently.

In short conclusion, theoretically, it is the most appropriate method for estimating VaR for non-linear portfolios.

4.- Theoretical Comparison

Laying aside methods based on the Extreme Value Theory, the comparison focuses on the three classics methods: variance-covariance matrix (delta and delta-gamma approaches), historical simulation and Monte Carlo simulation. In a more general level, the discussion centres on the choice between the parametric methods (delta and delta-gamma) or the simulation method (historical and Monte Carlo). Theoretically and very briefly, the analytic method is both easier to implement and faster, whereas the simulation methods (historical and Monte Carlo), being more difficult, and requiring a bigger computational effort, indeed are more exact in case of complex portfolios.

Therefore, the first conclusion we draw, is that **a best VaR estimation method, in general or absolute terms does not exist.** On the contrary, **the choice** of one or the other **depends,** among others aspects less relevant for our subject, **on the kind of portfolio** for which we wish to estimate the VaR.

As to the kind of portfolio, we need to highlight three factors:

a) The distributions of the individual instruments in the portfolio;
b) The nature and extent of any kind of non-linearity that may exist in the portfolio and
c) The particular ways in which each risk factor interacts, affecting the overall portfolio return.

If the portfolio complies with the underlying assumptions of the variance-covariance matrix method are present in the portfolio, then this will be the best VaR estimation procedure for such a portfolio; since applying another (for instance, Monte Carlo simulation) would not give more accuracy, and would have higher cost and need more time for the estimation.

Although the individual distributions of each one of the portfolio instruments are not usually normal, due to the Central Limit Theorem, assuming normality in the portfolio distribution is less restrictive. We saw this in the specific case of an options portfolio, providing that size were higher than twenty and that the options were absolutely independent (previous epigraph). What is already more restrictive is the fact of assuming linearity when there is not (even assuming a quadratic approximation).

Therefore, theoretically the choice of one or another VaR estimation method depends, especially, on whether the portfolios are linear or non-linear:

If the portfolios are lineal, the accuracy of VaR estimates with the variance-covariance matrix approach is as good as the one obtained with simulation methods. Besides, the estimation is easier and faster with the analytic method.

If the portfolios are non-linear, the variance-covariance matrix method provides poorly accurate VaR estimates. In this case, the best method would be Monte Carlo simulation, for although it is more complex (and theoretically slower), the VaR accuracy is higher.

We can see how, from a theoretical point of view, **the comparison of different VaR estimation methods** (even already focused on the specific case of non-linear portfolios) **forces a choice between the precision of the VaR and the rapidity of such estimation**.

Regarding this trade-off (accuracy versus computational time), we must mention that:

1. First, in real life, the biggest effort, both computational and in time, which is related to the Monte Carlo simulation method is attributed, is a bit exaggerated. Besides, due to the technological steps forward in the field of the information systems, this effort will be diminished gradually.

2. Second even if this “trade-off” were true, we could not prefer calculation speed over precision, especially in the case in which we have framed the VaR survey, the banking crisis prediction one or the medium-term control of banks solvency. A trader, while working in real time, could prefer quickness to precision, but a risk supervisor-manager, either internal or external, could not. We must not forget that the required bank capital amount to face market risk, depends on the VaR number, calculated by the bank. Moreover, due to the big

interest that the VaR has arisen, it is not farfetched to state that, working in real time with Monte Carlo simulation will not be contradictory in a short time, not even theoretically. Furthermore, we must here remember the works and techniques that allow solving the speed drawback of this method, such as: a) the Variance Reduction Techniques (we refer directly to DOWD (1998), p.9, who makes an analysis using the recent literature about this subject); b) Quasi-Monte Carlo Method: proposed by PASKOV and TRAUB (1995); and c) Scenario Simulation Methodology, proposed by JAMSHIDIAN and ZHU (1997). The first study that applies this proposal is the one by HULL and WHITE (1998).

In conclusion, from a theoretical point of view, the choice of the most convenient VaR estimation method depends on the kind of portfolio and on the use given to the VaR.

In the case of the subject we are dealing with (VaR calculation of actual non-linear portfolios in banking solvency supervision context), the accuracy given by the Monte Carlo simulation method must be preferred to the quickness of the variance-covariance matrix method.

In any case, it is advisable to find empirical evidence that could support such a theoretical conclusion.

And we have to point out that, in spite of the abundant literature revised, empirical literature is comparatively scarce. We have not found any study dealing with the obtaining of such evidence in the case of **non-linear actual** portfolios, which is the one we propose in this research. **The novelty and importance of this study lies, therefore, in this fact**.

PRITSKER (1997) is the only author who compares empirically every existent delta and delta-gamma methods, and two of Monte Carlo simulation (*Modified Grid* Monte Carlo and Monte Carlo *with full repricing*). He does not compare the historical simulation method. Besides, he conducts his work for different portfolios of currency options. The only “shortcoming”, as he himself recognises, is that he does not deal with actual portfolios.

When concluding, the author himself recognises the two most restrictive assumptions of his study and encourages to the repetition of this research without them. That has been our aim, by excluding one of them (we work with actual portfolios of options and, besides, hedged portfolios):

"*More importantly, the errors of VaR estimation methods when actually used are likely to be different than those reported here for two reasons: (...) Second, firms portfolios contain additional types of instruments not considered here, and* ***many firms have hedged positions****, while the ones used here are unhedged. In order to better characterise the method accuracies and computational time,* ***it is important to do the type of work performed here using*** *more realistic factor shocks, and* ***firm's actual portfolios***".

<table>
<tr><td rowspan="14">Classic Approach: Based on the probability distribution of returns</td><td rowspan="6">Delta Approach:
(only applicable theoretically to linear or quadratic positions)</td><td rowspan="6">Variances-Covariances Matrix (Parametric and Analytic)</td><td colspan="2">DELTA METHOD (CORRELATION METHOD[12] ó DELTA-NORMAL METHOD[b])</td></tr>
<tr><td rowspan="5">DELTA-GAMMA METHODS</td><td>DELTA-GAMMA WiLSON (DELTA-GAMMA MINIMIZATION[c] or GAMMA-NORMAL) [d]</td></tr>
<tr><td>DELTA-GAMMA-DELTA</td></tr>
<tr><td>DELTA-GAMMA JOHNSON</td></tr>
<tr><td>DELTA-GAMMA CORNISH-FISHER</td></tr>
<tr><td>DELTA-GAMMA MONTECARLO[h]</td></tr>
<tr><td rowspan="8">Global Approach:
(applicable to both linear and non-linear positions)</td><td rowspan="7">Non Parametric</td><td colspan="2">HISTORICAL SIMULATION (EMPIRICAL)</td></tr>
<tr><td rowspan="6">MONTECARLO SIMULATION
(with Bootstraping)</td><td>ESTRUCTURED MONTECARLO [e] (FULL MONTE CARLO METHOD) [f]</td></tr>
<tr><td>QUASI-MONTE CARLO</td></tr>
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<tr><td>MODIFIED GRID MONTE CARLO [g]</td></tr>
<tr><td>DELTA-GAMMA-MONTECARLO [h]</td></tr>
<tr><td></td></tr>
<tr><td>Parametric</td><td colspan="2">MONTE CARLO SIMULATION</td></tr>
<tr><td>EVT Approach: Based on the probability distribution of the extreme returns</td><td colspan="4">Extreme Value Theory Based Methods</td></tr>
</table>

Table 1.- Different VaR estimation methods
Source: Own elaboration

[a]. Allen (1994); [b]. Wilson (1994); [c]. Pritsker (1997); [d]. Fallon (1996); [e]. Longerstaey et al. (1996); [f]. Pritsker (1997); [g]. Pritsker (1997); [h]. Pritsker (1997).

4. EMPIRICAL STUDY: COMPARING DIFFERENT VAR ESTIMATION METHODS WITH ACTUAL BANK NON-LINEAR PORTFOLIOS OF CURRENCY OPTIONS.

In this study, we try to find empirical evidence that supports our theoretical conclusion.

We compare, for that, the two important VaR estimation methodologies in existence: the analytic one of the variance-covariance matrix and the Monte Carlo simulation.

We work with a non-linear actual portfolio: more specifically with the portfolio of currency options on 21st October 1996, of one of the big Spanish banks.

We calculate the VaR of such a portfolio, using each of the mentioned methods and we analyse the results obtained by their comparison.

Now, we are going to proceed describing our empirical study: data and methodology used and the analysis of the obtained results.

4.1.- DATA:

All the data used in this research are **ACTUAL:** portfolio, interest rates (monthly, 3 monthly, 6 monthly, and yearly), volatilities, correlations, etc.

4.1.1.- Description of the currency options portfolio

We use a portfolio of currency options of one of the big Spanish banks, for 21st October 1996; a hedged portfolio (with delta hedging) which, consequently, includes both options positions and spot positions (the delta hedging).

Of course, the decision about the kind of portfolio used in our study has not been either random or irrelevant, but rather on the contrary. The choice has been based on two facts:

1. No such study had been made until now. The only study that has compared the different estimation methods with portfolios of options did not use actual portfolios. The author of this study recognised that this was precisely the most restrictive assumption of his research, and as he did not work with actual portfolios, he did not take into account the effect of hedging positions
2. Once the choice regarding an actual portfolio and portfolio of options was made, we thought that the most appropriate underlying asset would be currencies, because in this case, the phenomenon of fat tails is specially significant, increasing the difficulty of the study.

Due to the necessary confidentiality (a requirement of the bank that provided us its portfolio), we do not include[13] such a portfolio; but we describe it hereinafter.

[13] A more detailed description of the actual portfolio used may be obtained from mcoronado@cee.upco.es on request

We will avoid repeating in words the information contained in the Tables. However it is right to point out some of the characteristics of this portfolio:

- The large majority of the cash positions correspond to the delta hedging. The rest appear as a result of the fact that some options (very few) reach their expiration date and are exercised.

- For the options classified as ATM (at-the-money), ITM (in-the-money) and OTM (out-of-the-money), we have considered the following:

  ✓ If its delta is less than 47% ($\lambda < 47\%$), the option is considered as OTM.
  ✓ If its delta is between 47% and 53% ($47\% \leq \lambda \leq 53\%$), it is considered that the option is ATM.
  ✓ If its delta is more than 53% ($\lambda > 53\%$), then the option is ITM.

  That is to say, it has been considered that ATM were, not only the options with λ= 50%, but also those whose delta is in the range 47%-53%, or in other words, using the criteria employed in the markets. (It is very difficult that the option had exactly a delta of 50%).

- It is necessary to remember that when the portfolios are near the money (ATM) and whose time to maturity is short (one week or less), the variance-covariance matrix method (the delta-gamma method) works "worse", in the sense that it is when the non-linear characteristics of the options are worse recognised, and consequently, when the differences are larger between the VaR estimations obtained with this method and the Monte Carlo simulation.

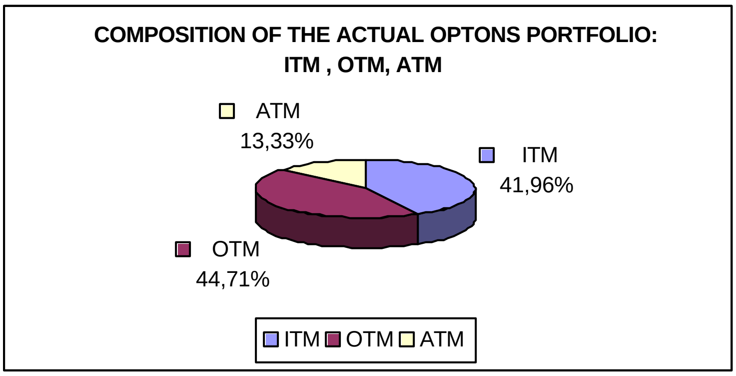
  In our portfolio, exactly the opposite occurs, as can be deduced from the **Graphics 1[14] and 2**, respectively. In our portfolio, there are very few options at-the-money (ATM) or close to the money, and for a large majority of the options, the time to maturity is very long. No option exists whose time to maturity is one week, nor even 15 days; in the entire portfolio, there is only one option with time to maturity of 21 days; the rest has a very long off expiration date

  That is to say, precisely the least favourable characteristics to obtain the results that, initially and theoretically, we thought would be produced (a large difference between the VaR results obtained with the analytic method and with the Monte Carlo simulation method). Thus, a priori, in our portfolio, the delta-gamma approach would work better than for other options portfolios, capturing better the non-linear characteristics of the options; therefore, it is possible to think that the differences between both methods would not reveal themselves so clearly. Upon analysing the results, we will see that this is not so; but even in these conditions (the most unfavourable ones), the expected results were obtained (which give them even more validity).

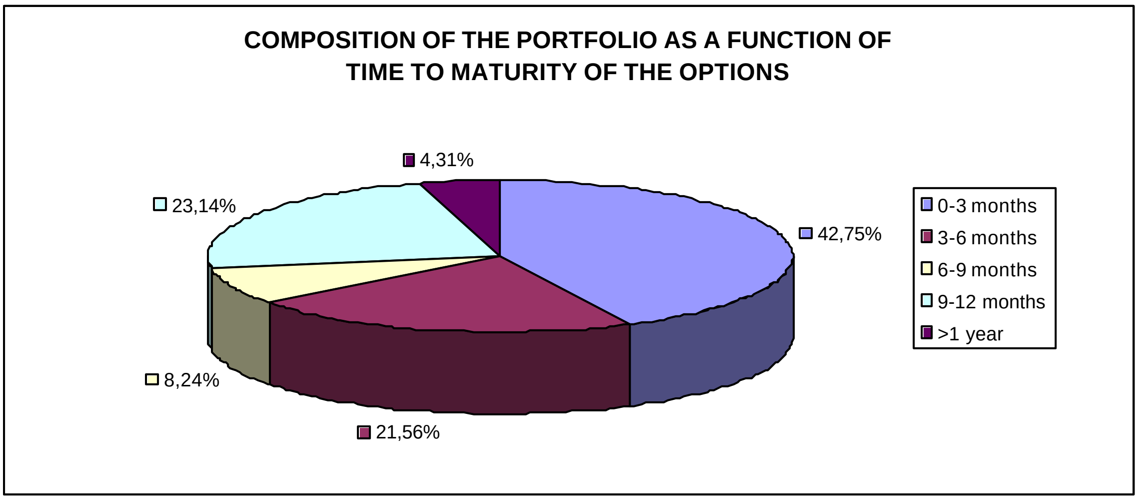
- For the options classification, according to their time to maturity, we considered years of 360 days.

[14] A detailed graphic description is available from mcoronado@cee.upco.es.

- The Garman-Kohlhagen model is the method that has been used to value the currency options.



Graphic 1



Graphic 2

4.1.2.- Variance-Covariance Matrix

To calculate the VaR, use has been made of the variance-covariance matrix which is provided, through the Internet, by J. P. Morgan and Reuters.

Therefore, it deals with volatilities and correlations, estimated through exponential weighted moving averages, with a decay factor of 0,94.

The variance-covariance matrix used can be obtained from mcoronado@cee.upco.es (in short, 25 volatilities and 325 correlations. In brief, they are daily volatilities and correlations for a confidence level of 95%).

4.2.- METHODOLOGY

With the above-mentioned data, we calculate the VaR of the **hedged portfolio**, at 21st October 1996 (that is to say, of the presented portfolio obtained “without more ado”) according to each one of methods presented: variance-covariance matrix method and Monte Carlo simulation method.

With this aim, the following software has been used: Excel, SPSS and Crystal Ball 4.0.

With each one of these methods, the VaR has been calculated for the different parameters:

1. Confidence level: specifically for 99%; 97,5%; 95% and 90%.
2. Time periods: one day, one week (5 trading days), a fortnight (10 trading days) and one month (20 trading days).

Moreover, when the VaR is calculated with the Monte Carlo Simulation Method, different simulations have been carried out each one with a different number of iterations. Specifically, the VaR calculations with the Monte Carlo method have been made with simulations of 10, 100, 500 and 1000 iterations (for each combination of confidence level and periods of time mentioned).

With this, we try to analyse the relationship between the number of iterations with the accuracy in the estimation and the computational time of such estimation.

Moreover, we made the same calculations for the unhedged portfolio; that is, excluding the part of hedging and keeping only the options positions themselves. With this, we are aim to obtain two things:

1. To analyse the risk difference (measured by the VaR) between the hedged portfolio (using delta hedging) and the unhedged one.
2. To show the importance of working with actual portfolios (which usually include the hedgings), as the conclusions obtained are more appreciated and are more realist than in the case than of the example of considering the isolated options (as do the authors who make the portfolios in an artificial way).

Regarding the methods that have been used the following should be said:

1. Analytic method of the variance-covariance matrix:
   We have used the RiskMetrics^{TM} method, with all its underlying assumptions, that we have already specified in part 3, among others, the use of the square root of time rule to calculate the VaR of different periods, starting from the daily VaR.
2. Monte Carlo Simulation Method:
   Here, a parametric Monte Carlo simulation method has been used, assuming that the stochastic process described by the prices is log-normal.

The same seed has been used in all the simulations, in particular 1, in order to allow the comparison among them.

And use has been made of the Cholesky decomposition for the correlations matrix.

As all the data that we have correspond exclusively to the portfolio situation on just one day (21st October 1996), we cannot follow up on the real losses that have occured in the portfolio (historical data), with the aim of validating the VaR estimates (according to anyone of the tests or validation systems of the VaR estimation methods used -backtesting-).

4.3.- EMPIRICAL STUDY RESULTS: <u>CONCLUSIONS</u>

Here the returns distributions (profit and loss) are obtained from the actual portfolio by means of the Monte Carlo simulation Method, for the different periods of time and number of iterations[15]. From these, the appropriate percentiles have been calculated for the already established confidence levels (99%; 97,5%; 95% and 90%).

Tables 2 (for the case of the hedged portfolio) and 3 (for the unhedged portfolio) show the VaR final **results**, as of 21st October 1996, obtained with each one of the methods and for the different confidence levels and periods of time. The differences of results between both methods are also shown in these tables. The four VaR results obtained by Monte Carlo Simulation, for each combination of period of time and confidence level (in the hedged portfolio as well as in the unhedged one), correspond in this order, to the different number of iterations used in the simulation: 10, 100, 500 and 1000 iterations.

From the analysis of the results of the empirical study, summarised in the Tables 2 and 3, we arrive at the following **<u>CONCLUSIONS</u>**:

1. The differences between the VaR estimations obtained with each one of the methods are much larger in the case of the hedged portfolio than in the unhedged one. This is so for all the confidence levels, periods of time and number of iterations.

   For instance, and just for the case of 1000 iterations, 97,5% of confidence level and one day, the difference between the VaR results of both methods is 10% for the hedged portfolio and just 1% for the unhedged one.

   Thus, it shows the need that empirical studies of such kind should be made with actual portfolios where, as it is obvious, the conclusions are much more clear and are not seen as distorted or impoverished by any kind of “artificial” choice.

2. The largest differences between methods (in both hedged and unhedged portfolios) are obtained for the VaR calculated with a 99% of confidence. That is to say, when we concentrate the tails of the probability distributions,

[15] Interested in that information please contact: mcoronado@cee.upco.es.

which is when the fat tails problems appear, due in part to a lack of normality or in part to a lack of linearity (which in turn, is a cause of non-normality) as it occurs in the kind of portfolio with which we are working.

3. In the case of the hedged portfolio (the real one), the biggest difference between both methods is obtained for the VaR with a confidence of 99% and for any period of time (the difference is about 12-13%); and the least difference is obtained for the daily VaR with a 90% probability (about 3% of difference).

The importance of this conclusion can be seen more clearly if we remember that, from the supervisors point of view (Basel in particular), the bank capital requirements for their market risk, are established according to the VaR measurement, calculated with a probability of 99% and 10 days; precisely where the differences are largest.
If it is about measuring the risk in a more accurate way, in order to control solvency, we have already proved that, in this case (options), the Monte Carlo Simulation Method captured better the non-linearity and, all things considered, the risk. But, bearing in mind that the VaR calculated with this method (and 1000 iterations) is 12% higher than the VaR calculated with the analytic method, the bank will have a tendency to use the analytic one. This is because a smaller amount of capital is required, benefiting its profitability; but it would not be controlling the solvency of that bank in a correct way.

4. In the hedged portfolio, and only comparing the analytic VaR with the Monte Carlo VaR with 1000 iterations, the first one is always less (for any confidence level) than the second one, calculated by simulation.

5. Of course, with both methods, the longer the period of time and/or the higher the confidence level, the larger the VaR estimates obtained.

6. Logically also, the VaR amounts obtained with both methods (and for all confidence levels, periods of time and number of iterations), are much bigger in the unhedged portfolio than in the hedged one. This reveals the delta-hedging effect in the reduction of the entire portfolio risk.
For instance, the 10 days analytic VaR with 99% of confidence, amounts to 156.1 million pesetas in the hedged portfolio and to 532.6 million in the unhedged one; or the daily analytic VaR with 99%, amounts to 49.4 million with the hedged and 168.4 million with the unhedged portfolio.

7. In fact, if we compare the analytic VaRs, which were obtained for the unhedged portfolio and those obtained for the hedged one, we can see a VaR decrease of approximately 70,7% (for all confidence levels and periods of time). That is to say, the incremental VaR of the hedging positions over the unhedged portfolio is negative, (or in other words, the risk drops and, consequently, the VaR) some 70,7%.

8. Of course the differences between methods become smaller at the same time that the number of iterations used in the Monte Carlo simulation increases (for any percentage, period of time and kind of portfolio: hedged or unhedged). For instance, in the hedged portfolio, for a confidence level of

99% and 20 days, the differences are: 64% in the case of 10 iterations; 27% for 100 iterations; 21% for 500 iterations and 12% when the iterations reache 1000.

9. We have also analysed the theoretical opinion of some risk managers, who were interviewed regarding the differences between methods**.** These differences would be reduced if the period of time for which the VaR is calculated is reduced, and would be very small in the case of daily VaR because the delta hedging would take up almost the whole risk. This opinion does not find empirical support in our study; perhaps even the contrary is the case. For example, if we concentrate on only the hedged portfolio and at 99% and 1000 iterations, the differences would be 12% or 13% for all the periods (20 days, 10 days, 5 days and 1 day). Even, if we take other confidence levels, the difference increases slightly as the same time as the period of time decreases.

10. As a final commentary, we would like to point out that, in our study, we have not observed so clearly the need to choose in the trade-off accuracy versus computational time. The slowness of the calculations that is attributed to the Monte Carlo simulation method is not so exaggerated, (considering that we have worked with a low powered PC, etc). That is to say, the precision advantage in VaR estimates obtained with Monte Carlo simulation is not undermined to the point of being annulled, because the slowness of calculating such an estimation.

Therefore, in those cases in which the accuracy of the VaR estimation takes priority over the speed of estimation (as for instance, the supervision of banking solvency), the most accurate method must be chosen (in our case, the Monte Carlo simulation method). Moreover, we do not think that we are sacrificing something so important as the time, because it is not so evident.

If the context of the comparative study were another, for example, the daily activity of trading, in this case, speed could take priority over the accuracy of the estimation; however, we insist that we should measure carefully such time saving, that theoretically, it is produced with the other method and we should analyse the suitability or not of the institution investing on the development of these new techniques and methods that are improving, in a spectacular way, the speed of the Monte Carlo Method

HEDGED PORTFOLIO VAR AS OF 21 OCTOBER 1996 (IN MILLION PESETAS)													
		CONFIDENCE LEVEL											
		99%			97.5%			95%			90%		
		METHOD		Difference (%)	METHOD		Difference (%)	METHOD		Difference (%)	METHOD		Difference (%)
		Analytic	Monte Carlo		Analytic	Monte Carlo		Analytic	Monte Carlo		Analytic	Monte Carlo	
HORIZON PERIOD	20 DAY	220.8	134.8	64%	186.0	134.8	38%	156.1	134.8	16%	121.6	134.8	-10%
			173.3	27%		146.0	27%		138.1	13%		129.1	-6%
			183.0	21%		158.4	17%		141.4	10%		120.8	1%
			197.1	12%		173.3	7%		145.8	7%		113.4	7%
	10 DAYS	156.1	94.8	65%	131.5	94.8	39%	110.4	94.8	16%	86.0	94.8	-9%
			121.5	29%		102.2	29%		99.5	11%		90.6	-5%
			129.1	21%		111.5	18%		100.1	10%		85.4	1%
			138.5	13%		121.5	8%		103.0	7%		83.9	2%
	5 DAYS	110.4	66.7	65%	93.0	66.7	39%	78.1	66.7	17%	60.8	66.7	-9%
			85.3	29%		71.9	29%		70.1	11%		63.9	-5%
			91.1	21%		79.8	17%		71.5	9%		60.4	1%
			97.7	13%		85.3	9%		72.6	8%		59.3	3%
	1 DAY	49.4	29.6	67%	41.6	29.6	40%	34.9	29.6	18%	27.2	29.6	-8%
			37.7	31%		32.7	27%		31.5	11%		29.4	-8%
			40.5	22%		35.2	18%		31.7	10%		27.0	1%
			43.7	13%		37.9	10%		32.5	7%		26.5	3%

Table 2. VaR Empirical Estimation of the hedged portfolio, 21st October 1996 (in millions pesetas), according to the Analytic Method and the Monte Carlo Simulation Method; for different confidence levels and periods of time. Difference between the results obtained with each method (expressed in percentage).).
Source: Own elaboration

UNHEDGED PORTOLIO VAR AS OF 21 OCTOBER 1996 (IN MILLION PESETAS)													
		CONFIDENCE LEVEL											
		99%			97.5%			95%			90%		
		METHOD		Difference (%)	METHOD		Difference (%)	METHOD		Difference (%)	METHOD		Difference (%)
		Analytic	Monte Carlo		Analytic	Monte Carlo		Analytic	Monte Carlo		Analytic	Monte Carlo	
HORIZON PERIOD	20 DAYS	753.2	572.0	32%	634.6	572.0	11%	532.5	572.0	-7%	414.9	572.0	-27%
			572.01	32%		562.6	13%		503.6	6%		424.1	-2%
			707.7	6%		620.1	2%		502.9	6%		378.1	10%
			720.1	5%		646.6	-2%		523.4	2%		401.9	3%
	10 DAYS	532.6	402.2	32%	448.7	402.2	12%	376.6	402.2	-6%	293.4	402.2	-27%
			402.2	32%		394.7	14%		353.6	6%		298.0	-2%
			505.1	5%		436.3	3%		353.5	7%		267.3	10%
			506.8	5%		451.9	-1%		367.8	2%		283.2	4%
	5 DAYS	376.6	283.2	33%	317.3	283.2	12%	266.3	283.2	-6%	207.5	283.2	-27%
			283.2	33%		277.4	14%		248.7	7%		209.8	-1%
			350.7	7%		307.2	3%		248.9	7%		189.2	10%
			357.5	5%		317.6	0%		259.1	3%		199.6	4%
	1 DAY	168.4	125.9	34%	141.9	125.9	13%	119.1	125.9	-5%	92.8	125.9	-26%
			160.0	5%		123.0	15%		113.4	5%		93.2	0%
			156.0	8%		136.4	4%		110.6	8%		84.7	10%
			158.7	6%		140.8	1%		115.2	3%		88.1	5%

Table 3. VaR Empirical Estimation of the unhedged, 21st October 1996 (in millions pesetas), according to the Analytic Method and the Monte Carlo Simulation Method; for different confidence levels and periods of time.
Difference between the results obtained with each method (expressed in percentage).
Source: Own elaboration

5. REFERENCES

ALLEN, Michael (1994): Building a Role Model. *Risk*, Vol. 7, Nº 9, (September), pp. 73-80.

BARONE-ADESI, G. et al. (1998): VaR without correlations for Nonlinear Portfolios. University of Alberta, Canada

BARONE-ADESI, G. and **GIANNOPOULOS**, K. (1996): A simplified Approach to the conditional estimation of VaR. *Futures & Options World* (October), pp. 68-72.

BASLE COMMITTEE ON BANKING SUPERVISION (1996a): Amendment to the Capital Accord to Incorporate Market Risks. *Basle Committee on Banking Supervision*, Basle, (January).

BASLE COMMITTEE ON BANKING SUPERVISION (1996b): Supervisory Framework for the Use of Backtesting in Conjunction with the Internal Models Approach to Market Risk Capital Requirements. *Basle Committee on Banking Supervision*, Basle, (January).

BASLE COMMITTEE ON BANKING SUPERVISION (1995): An Internal Model Based Approach to Market Risk Capital Requirements. *Basle Committee on Banking Supervision*, Basle, (April).

BEDER, T (1995): VaR: Seductive but Dangerous. *Financial Analysts Journal*, (September/October), pp. 12-24

BEDER, T. (1996): Report Card on Value at Risk: High Potential but Slow Starter. *Bank Accounting & Finance*, Vol. 10, Nº 1 (Fall), pp.14-25.

CORONADO, María (2000a): Extreme Value Theory (EVT) for Risk Managers: Pitfalls and Opportunities in the Use of EVT in Measuring Value-at-Risk (VaR). *Proceedings of the VIII Foro de Finanzas*, Madrid, October 25-27, 2000; and *Proceedings of the V Congreso Nacional and III Hispano-Italiano de Matemática Financiera y Actuarial*, Bilbao, April 26-28, 2000.

CORONADO, María (2000b): La Ética de los mercados de derivados financieros: Lecciones a aprender de las pérdidas con derivados. *Boletín de Estudios Económicos*, Vol. LV, Nº 170, pp. 273-302, Agosto de 2000. Universidad Comercial de Deusto, Bilbao (España).

CORONADO, María (1999a): Valor en Riesgo: Aplicación a Carteras de Opciones en Entidades Financieras. *Actualidad Financiera*, Nº 11, Noviembre, pp. 37-65.

CORONADO, María (1999b): Predicción de Crisis Bancarias: el Value-at-Risk (VaR) como Medida del Riesgo de Mercado. (Bank Failures Prediction: VaR as a Market Risk Measure, PhD Dissertation) Tesis Doctoral, Universidad P. Comillas de Madrid (April).

DOWD, K. (1998): *Beyond Value at Risk The New Science of Risk Management*. Ed. John Wiley and Sons, London.

ESTRELLA, A. (1996): Taylor, Black and Scholes: Series Approximations and Risk Management Pitfalls. *in Risk Measurement and Systemic Risk, Proceedings of a Joint Central Bank Research Conference*. Board of Governors of the Federal Reserve System (Ed.), Washington, D.C. , pp. 359-379.

FALLON, W. (1996): Calculating Value-at-Risk. Working Paper Nº 96-49, Wharton Financial Institutions Center's Conference on Risk Management in Banking, (October 13-15).

FINGER, C. (1997): When is a Portfolio of Options Normally Distributed? *Quarterly RiskMetrics™ Monitor*, Third Quarter (September 15), pp. 33-41.

GIZYCKI, M. and **HEREFORD**, N. (1998): Assessing the Dispersion in Bank's Estimates of Market Risk: The Results of a Value-at-Risk Survey. Discussion Paper 1 (October). Australian Prudential Regulation Authority.

GROUP OF THIRTY, THE (1993): Derivatives: Practices and Principles. *Global Derivatives Study Group*, G-30 Report, (July).

HENDRICKS, Darryll (1996): Evaluation of Value-at-Risk Models Using Historical Data. *Economic Policy Review, Federal Reserve Bank of New York*, (April), pp. 39-70.

HULL, J. C. and **WHITE**, A. (1998): Value at Risk when Daily Changes in Market Variables are not Normally Distributed. *Journal of Derivatives*, Vol. 5, Nº 3, (Spring), pp. 9-19.

JACKSON, P.; et al. (1997a): Bank Capital and Value at Risk. *The Journal of Derivatives*, Vol. 4, (Spring), pp. 73-89

JAMSHIDIAN, F. and **ZHU**, Y. (1997): Scenario Simulation: Theory and Methodology. *Finance and Stochastics*, Vol. 1, (January), pp. 43-67

JORION, P (1995): Predicting Volatility in the Foreign Exchange Markets. *Journal of Finance*, Vol. 50, Nº 2, (June), pp. 507-528.

JORION, P. (1997): *Value at Risk. The New Benchmark for Controlling Derivatives Risk*. Ed. Irwin, Chicago.

LONGERSTAY, J et al. (1996): *RiskMetrics™. Technical Document.* Morgan Guaranty Trust Company, Fourth Edition, New York (December 17).

MARSHALL, C and **SIEGEL**, M. (1997): Value at Risk: Implementing a Risk Measurement Standard. *The Journal of Derivatives*, Vol. 4, (Spring), pp. 91-110.

MINNICH, M. (1998): A Primer on Value at Risk. In Fabozzi, Frank J. (ed.): *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (1998); Chapter 3, pp. 39-50.

PASKOV, S. H. and **TRAUB**, J. F. (1995): Faster Valuation of Financial Derivatives. *The Journal of Portfolio Management*, Vol. 22, Nº 1, (fall), pp. 113-120.

PRITSKER, M. (1997): Evaluating Value-at-Risk Methodologies: Accuracy versus Computational Time. *Journal of Financial Services Research*, (October/December), pp. 201-241.

SCHACHTER, B. (1995): Comments on Taylor Black and Scholes: Series Approximations and Risk Management Pitfalls by Arturo Estrella. Manuscript, Office of the Comptroller of the Currency, (September).

VENKATARAMAN, S. (1997): Value at Risk for a Mixture of Normal Distributions: The Use of Quasi-Bayesian Estimation Techniques. *Economic Perspectives, Federal Reserve Bank of Chicago*, Vol. 21, Nº 2, (March-April), pp. 2-13.

WILSON, Thomas C. (1994): Plugging the Gap. *Risk*, Vol. 7, Nº 10, (October), pp. 74-80.