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VALUE-AT-RISK AND MIXTURE DISTRIBUTIONS

Summary:

From time to time, participants in the bond and equity markets notice that price relationships between various securities do not hold to their previous patterns. This is most often seen at times of crisis for a particular sector of the market. During these times, value-at-risk analysis and other risk management methodologies based upon the normal relationships may not provide an accurate picture of the actual risk to the portfolio. In this article, we will explore "mixture distributions," a combination of two distinct distributions that better define the relationship between different securities.

One of the principles of portfolio management is diversification. Investment choices are made, at least partially, based upon historical relationships between different securities in an attempt to select a group of securities whose prices do not move similarly during various market events. Such a portfolio is assumed to be less risky due to this diversification.

However, investors recognize that these historical relationships do not always hold. The most common examples are the various shifts that occur during times of high volatility in the stock market and the "flight to quality" impact on the U.S. Treasury market. The collapse in the Asian markets provides a recent example, where foreign debt prices fell dramatically as Treasury prices increased. Investors who had hedged the interest rate exposure of their foreign debt holdings with Treasuries found themselves losing money on both the asset and the hedge.

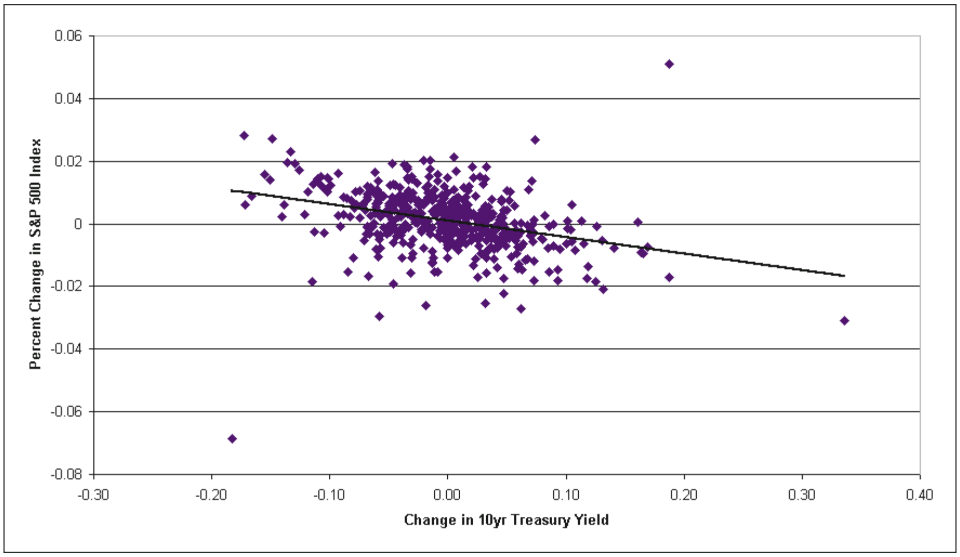
In this article, we explore this issue by examining the relationship between two asset types, the S&P 500 index and the 10-year Treasury note. Our goal was to determine if the relationship between these assets could be better defined by two separate mathematical relationships rather than a single regression equation.

Analysis

Daily percentage price changes of the S&P 500 index since 1996 are graphed against daily yield changes of the 10-year Treasury in Figure 1. The slope of the regression line was calculated to be -.052. Thus, for every 1 basis point increase in the 10-year Treasury yield, we would expect a -.052% change in the S&P 500 index. However, this relationship is less than perfect (The correlation is -.33.), and many of the points do not fall on the regression line. The difference between these data points and the regression line represents what we generally think of as basis or spread risk.

Looking at the graph, there is a recognizable pattern to the data points. The data points form a rough oval around the point representing no change in yield or price for both securities. However, there are a few points that fall a significant distance from the regression line and are clearly not part of the pattern. After making the assumption that these points represent a separate distribution, our next step was to isolate these points mathematically from the others.

Figure 1: Change in S&P 500 Index vs. Change in 10-Year Treasury Yield



The slope of the regression line was used to calculate the residual difference between the expected and actual price changes of the S&P index as a function of the change in the 10-year Treasury yield. The standard deviation of the residuals was then calculated and each residual expressed as a standard deviation. The data points were then sorted into two groups: those with a low standard deviation residual and those with a high standard deviation residual. Several cutoff points were tested and the optimal division was determined to be three standard deviations. Table 1 compares the correlation coefficients (a measure of predictive ability) for the 2.5 and three standard deviation results. The graph of the two distributions using the three standard deviation cutoff is shown in Figure 2.

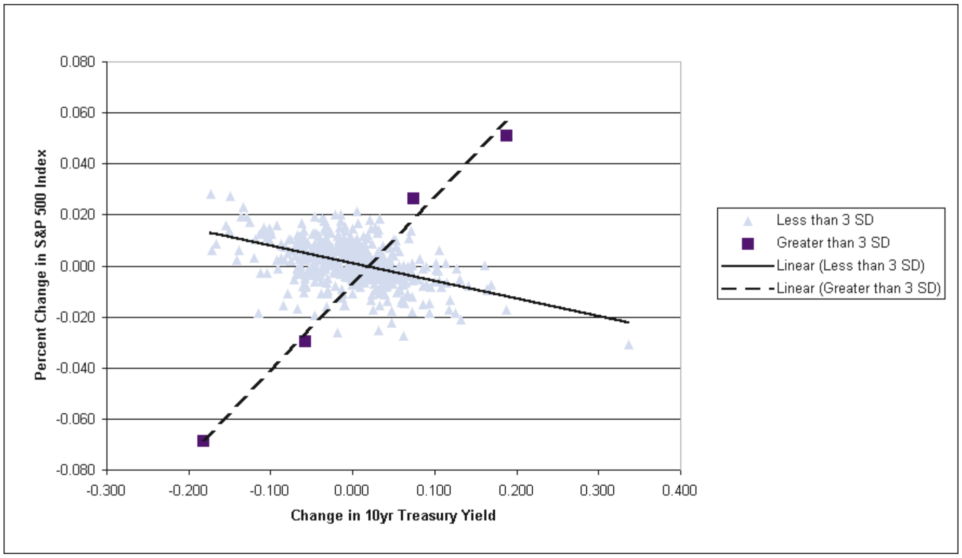
Table 1: Comparison of 2.5 and 3 Standard Deviation Cutoff Points

	10yrTreasury - S&P 500 Correlation	
	2.5 SD Cutoff	3 SD Cutoff
All Data	-.33	-.33
Low SD Residual	-.50	-.48
High SD Residual	.85	.99

By dividing the original distribution into two distributions using three standard deviations as the cutoff point, we have increased the predictive capabilities between 10-year Treasury yields and the S&P 500 Index. The original -.33 correlation becomes a -.48 correlation for yield movements less than three standard deviations. For yield movements greater than three standard deviations, the relationship completely changes to the opposite of the expected with a high positive correlation of .99. Rather than the S&P index increasing in value as Treasury yields decrease, the S&P index decreases when yields decrease and vice versa.

The impact on value-at-risk (VaR) calculations can be significant. In Table 2, we show the one standard deviation VaR calculations for a sample portfolio consisting of a $10 million long position in the S&P 500 and a $10 million short position in the 10-year Treasury note. For the analysis, we calculated the standard deviation for each of the distributions and used the correlation coefficients previously calculated and shown in Table 1.

Figure 2: Percent Change in S&P 500 Index vs. Change in 10-Year Treasury Yield



These calculations show that the VaR for less than three standard deviation movements in the S&P 500 index is significantly less than for all of the data. This was expected since we have assigned the outlying (high standard deviation) points to another distribution. What is surprising is the magnitude of the VaR for the greater than three standard deviation movements. Because of the high positive correlation of this distribution, the individual VaR calculations for the S&P 500 and the 10-year Treasury are nearly additive rather than partially offsetting.

Table 2: Value-at Risk Calculations

	One Standard Deviation Shift 10yr Trsy	S&P 500	Correlation	Portfolio VaR	Distribution Probability
All Data	44,784	95,510	-.33	91,358	1.000
<3 SD Residual	43,960	85,933	-.48	75,669	.992
>3 SD Residual	121,273	542,128	.99	662,788	.008

How do these results impact a VaR calculation? The main impact will, of course, be on calculations where a large confidence interval is used. For example, the 99% confidence interval for the entire distribution was calculated to be ($212,543), meaning that 99% of the time, portfolio losses are expected to be less than $212,543. Using the distribution probability weightings and an iterative process to determine the expected loss, the 99% confidence interval for the mixture distribution is ($186,175). However, using a 99.8% confidence interval, the expected maximum loss for the entire distribution is $262,927, while for the mixture distribution it is $446,000.

Conclusion

The implication for portfolio management is clear. Although price/yield movements greater than three standard deviations are low probability events, they can have a devastating impact on the portfolio. In our example portfolio, the expected hedge effects between the two securities vanished and the VaR was greatly increased. As a point of interest, 3 of the 4 observations in this distribution fell within an 8-day period in October and November 1997 when the Asian markets were collapsing. The fourth observation was in January 1998. Thus, although the events have a low probability of occurrence, a cluster of events such as this can be quite problematic.

Additional analysis should be completed to look for similar relationships in other asset types. By defining these relationships as separate distributions, portfolio managers gain a better understanding of their securities in times of high volatility, which would allow for more effective risk management.

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Quantitative Analysis

FGLMC 30 Year			MARKET February Settlement 1/23/98		SPREAD TO			OPTIONS ANALYSIS			RISK		
CPN	WAM	AGE	Price	Yield	WAL TRSY	Zero Curve	Fwrd Curve	OAS	Option Cost	Fwd Curve Effect	Mod Dur	Eff Dur	Convex
6.0	341	19	96-12	6.65	98	93	92	65	27	1	6.2	5.1	-0.7
6.5	354	6	98-25	6.75	106	103	102	61	41	1	6.4	4.5	-1.3
7.0	355	5	100-28	6.86	128	123	120	59	61	3	4.9	3.5	-1.7
7.5	355	5	102-12	6.92	140	130	135	69	65	-3	3.8	2.8	-1.5
8.0	352	8	103-12	6.84	136	121	130	73	58	-9	2.8	1.9	-1.2
8.5	352	8	104- 7	6.75	132	109	120	74	46	-11	2.4	1.3	-0.8
9.0	335	25	106-20	6.18	76	52	69	33	36	-16	2.4	1.1	-0.3
9.5	321	39	107-15	6.61	117	104	119	94	25	-14	2.6	1.7	-0.1
10.0	268	92	108-23	6.58	114	108	123	117	6	-14	2.6	2.0	0.2

		PREPAYMENT				PREPAYMENT DURATION				TSY HEDGE POSITIONS			
CPN	Price	Base PSA	FWD PSA	Base WAL	FWD WAL	Scale	Steep	Slide	Burn	2 Year	5 Year	10 Year	30 Year
6.0	96-12	124	120	9.3	9.5	0.18	-0.30	0.05	0.00	0.64	0.24	0.26	0.08
6.5	98-25	130	125	9.8	10.0	0.07	-0.24	-0.21	0.00	0.84	0.22	0.22	0.06
7.0	100-28	228	150	6.8	9.1	-0.04	-0.22	-0.52	-0.01	1.01	0.15	0.15	0.03
7.5	102-12	332	262	5.1	6.2	-0.15	-0.18	-0.68	0.00	1.00	0.13	0.09	0.03
8.0	103-12	480	405	3.5	4.1	-0.26	-0.20	-0.87	0.00	0.93	0.09	0.03	0.01
8.5	104- 7	606	531	2.8	3.2	-0.37	-0.26	-0.94	0.00	0.81	0.07	0.00	0.00
9.0	106-20	499	454	2.8	3.1	-0.61	-0.37	-1.28	-0.01	0.66	0.12	-0.02	-0.01
9.5	107-15	444	408	3.2	3.5	-0.58	-0.39	-1.00	-0.07	0.66	0.18	0.03	0.00
10.0	108-23	444	415	3.1	3.3	-0.59	-0.43	-0.79	-0.42	0.64	0.23	0.05	-0.01

GNMA 30 Year			MARKET February Settlement 1/23/98		SPREAD TO			OPTIONS ANALYSIS			RISK		
CPN	WAM	AGE	Price	Yield	WAL TRSY	Zero Curve	Fwrd Curve	OAS	Option Cost	Fwd Curve Effect	Mod Dur	Eff Dur	Convex
6.0	339	21	96-28	6.55	88	83	80	47	34	3	6.4	6.0	-0.8
6.5	348	12	98-24	6.76	109	105	104	61	43	2	6.2	5.3	-1.2
7.0	354	6	100-27	6.90	125	122	121	67	54	1	6.0	4.4	-1.7
7.5	353	7	102-15	7.06	146	141	143	75	68	-1	5.2	3.3	-2.1
8.0	353	7	103-17	7.17	163	154	161	84	77	-5	4.1	2.1	-2.1
8.5	352	8	105- 3	6.70	123	103	123	58	64	-18	2.8	0.8	-1.4
9.0	329	31	107- 9	6.04	61	38	59	14	45	-20	2.5	0.5	-0.6
9.5	321	39	108-13	6.14	70	51	70	39	31	-18	2.6	0.9	-0.2
10.0	261	99	109-28	6.42	96	91	107	89	17	-15	2.9	1.8	0.1

		PREPAYMENT				PREPAYMENT DURATION				TSY HEDGE POSITIONS			
CPN	Price	Base PSA	FWD PSA	Base WAL	FWD WAL	Scale	Steep	Slide	Burn	2 Year	5 Year	10 Year	30 Year
6.0	96-28	120	109	9.4	9.9	0.10	-0.19	-0.01	0.00	0.70	0.19	0.34	0.11
6.5	98-24	136	124	9.2	9.7	0.03	-0.17	-0.17	0.00	0.83	0.21	0.29	0.08
7.0	100-27	155	142	8.9	9.3	-0.06	-0.15	-0.46	0.00	0.99	0.19	0.22	0.06
7.5	102-15	197	166	7.5	8.5	-0.14	-0.15	-0.80	-0.01	1.08	0.15	0.14	0.03
8.0	103-17	292	240	5.5	6.5	-0.23	-0.17	-1.16	-0.01	1.06	0.11	0.04	0.02
8.5	105- 3	492	404	3.4	4.1	-0.40	-0.23	-1.93	-0.01	0.96	0.07	-0.07	0.00
9.0	107- 9	469	419	3.0	3.4	-0.64	-0.33	-1.98	-0.02	0.71	0.10	-0.08	-0.01
9.5	108-13	459	420	3.0	3.4	-0.70	-0.37	-1.64	-0.06	0.65	0.14	-0.04	-0.01
10.0	109-28	402	373	3.4	3.7	-0.64	-0.30	-1.13	-0.33	0.64	0.23	0.02	-0.01

Based on Andrew Davidson & Co., Inc. OAS model and prepayment model