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Noisy Time-Series Prediction using Pattern Recognition Techniques

Sameer Singh

{s.singh@exeter.ac.uk}

UNIVERSITY OF EXETER
DEPARTMENT OF COMPUTER SCIENCE
EXETER EX4 4PT
UNITED KINGDOM

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ABSTRACT

Time-series prediction is important in physical and financial domains. Pattern recognition techniques for time-series prediction are based on structural matching of the current state of the time-series with previously occurring states in historical data for making predictions. This paper describes a Pattern Modelling and Recognition System (PMRS) which is used for forecasting benchmark series and the US S&P financial index. The main aim of this paper is to evaluate the performance of such a system on noise free and Gaussian additive noise injected time-series. The results show that the addition of Gaussian noise leads to better forecasts. The results also show that the Gaussian noise standard deviation has an important effect on the PMRS performance. PMRS results are compared with the popular Exponential Smoothing method.

Keywords

Univariate time-series	Pattern Recognition	Noise injection
Computational Intelligence	Forecasting	System performance

I. INTRODUCTION

Time-series prediction is important in several domains. Delurgio (1998) summarises a large number of statistical techniques for predicting univariate time-series. The prediction of univariate variables is also useful in analysing multivariate systems. In the last decade, a large number of advanced methods of time-series prediction such as neural networks, genetic algorithms and other sophisticated computational methods have become popular (Azoff, 1994; Chorafas, 1994; MacDonald, and Zucchini, 1997). These different methods exhibit a certain degree of computational intelligence and perform better than others on specific problems. In forecasting systems such as neural networks, univariate predictions are based on previous observations and network architecture/ training algorithm must be optimised for accurate predictions. In statistical methods, the laws of prediction are more explicit and time-series must be analysed first to identify the correct model for prediction. One of the new developing methods of forecasting is through pattern imitation and recognition (Farmer, J. D. and Sidorowich, 1988; Motnikar et al., 1996). Such systems match current time-series states with historical data for making predictions. In this paper, a system based on this philosophy will be presented. It will be called the Pattern Modelling and Recognition System.

Noisy time-series are common in several scientific and financial domains. Noisy time-series may or may not be random in nature. The noise within a time-series signal could be identified using Fourier analysis (Brillinger, 1981). Conventionally, noise is regarded as an obstruction to accurate forecasting and several methods of filtering time-series to remove noise already exist. In this paper we take a different view. It is proposed that controlled addition of noise in time-series data can be useful for accurate forecasting. A number of studies on neural networks have reported superior network training on noise-contaminated data (Burton and Mpitos, 1992; Jim et al., 1995; Murray and Edwards, 1993). Fuzzy nearest neighbour methods for pattern recognition also perform better with noise (Singh, 1998). In this paper we will try to prove the hypothesis that noisy untreated time-series prediction under controlled conditions gives better results than prediction on original time-series. We also seek to identify these controlled conditions. The hypothesis will be tested on three benchmark series taken from the Santa Fe competition (Weigend and Gersehnfield, 1994) and the real US S&P index. We first describe a new pattern recognition technique for time-series forecasting using which the results have been produced.

II. PATTERN MODELLING AND RECOGNITION SYSTEM

The main emphasis of local approximation techniques is to model the current state of the time-series by matching its current state with past states. If we choose to represent a time-series as **y** = $\{y_1, y_2, ... y_n\}$, then the current state of size one of the time-series is represented by its current value y_n. One simple method of prediction can be based on identifying the closest neighbour of y_n in the past data, say y_j, and predicting y_{n+1} on the basis of y_{j+1}. This approach can be modified by calculating an average prediction based on more than one nearest neighbours. The definition of the current state of a time-series can be extended to include more than one value, e.g. the current state s_c of size two may be defined as $\{y_{n-1}, y_n\}$. For such a current state, the prediction will depend on the past state s_p $\{y_{j-1}, y_j\}$ and next series value y^+_p given by y_{j+1}, provided that we establish that the state $\{y_{j-1}, y_j\}$ is the nearest neighbour of the state $\{y_{j-1}, y_j\}$ using some similarity measurement. In this paper, we also refer to *states* as *patterns*. In theory, we can have a current state of any size but in practice only matching current states of optimal size to past states of the same size yields accurate forecasts since too small or too large neighbourhoods do not generalise well. The optimal state size must be determined experimentally on the basis of achieving minimal errors on standard measures through an iterative procedure.

We can formalise the prediction procedure as follows:

$$\ddot{y} = \phi(s_c, s_p, y^+_p, k, c)$$

where $\ddot{y}$ is the prediction for the next time step defined as a function ϕ of, s_c (current state), s_p (nearest past state), y^+_p (series value following past state s_p), k (state size) and *c* (matching constraint). Here $\ddot{y}$ is a real value, s_c or s_p can be represented as a set of real values, k is a constant representing the number of values in each state, i.e. size of the set, and *c* is a constraint which is user defined for the matching process. We define *c* as the condition of matching operation that series direction change for each member in s_c and s_p is the same.

In order to illustrate the matching process for series prediction further, consider the time series as a vector $\mathbf{y} = \{y_1, y_2, ... y_n\}$ where n is the total number of points in the series. Often, we also represent such a series as a function of time, e.g. $y_n = y_t$, $y_{n-1} = y_{t-1}$, and so on. A segment in the series is defined as a difference vector $\boldsymbol{\delta} = (\delta_1, \delta_2, ... \delta_{n-1})$ where $\delta_i = y_{i+1} - y_i$, $\forall i$, $1 \le i \le n-1$. A pattern contains one or more segments and it can be visualised as a string of segments $\rho = (\delta_i, \delta_{i+1}, ... \delta_h)$ for given values of i and h, $1 \le i,h \le n-1$, provided that $h > i$. In order to define any pattern mathematically, we choose to tag the time series y with a vector of change in direction. For this purpose, a value y_i is tagged with a 0 if $y_{i+1} < y_i$, and as a 1 if $y_{i+1} \ge y_i$. Formally, a pattern in the time-series is represented as $\rho = (b_i, b_{i+1}, ... b_h)$ where b is a binary value.

The complete time-series is tagged as $(b_1 ... b_{n-1})$. For a total of k segments in a pattern, it is tagged with a string of k *b* values. For a pattern of size k, the total number of binary patterns (shapes) possible is 2^k. The technique of matching structural primitives is based on the premise that the past repeats itself. Farmer and Sidorowich (1988) state that the dynamic behaviour of time-series can be efficiently predicted by using local approximation. For this purpose, a map between current states and the nearest neighbour past states can be generated for forecasting.

Pattern matching in the context of time-series forecasting refers to the process of matching current state of the time series with its past states. Consider the tagged time series $(b_1, b_i, ... b_{n-1})$. Suppose that we are at time *n* (y_n) trying to predict y_{n+1}. A pattern of size k is first formulated from the last k tag values in the series, $\rho' = (b_{n-k}, ... b_{n-1})$. The size k of the structural primitive (pattern) used for matching has a direct effect on the prediction accuracy. Thus the pattern size k must be optimised for obtaining the best results. For this k is increased in every trial by one unit till it reaches a predefined maximum allowed for the experiment and the error measures are noted; the value of k that gives the least error is finally selected. The aim of a pattern matching algorithm is to find the closest match of ρ' in the historical data (estimation period) and use this for predicting y_{n+1}. The magnitude and direction of prediction depend on the match found. The success in correctly predicting series depends directly on the pattern matching algorithm. The overall procedure is shown as a flowchart in Figure 1.

**Figure 1 here*

Figure 1 shows the implementation of the Pattern Modelling and Recognition System for forecasting. The first step is to select a state/pattern of minimal size (k=2). A nearest neighbour of this pattern is determined from historical data on the basis of smallest offset ∇. The nearest neighbour position in the past data is termed as "marker". There are three cases for prediction: either we predict high, we predict low, or we predict that the future value is the same as the current value. The prediction $\ddot{y}_{n+1}$ is scaled on the basis of the similarity of the match found. We use a number of widely applied error measures for estimating the accuracy of the forecast and selecting optimal k size for minimal error.

Mean Square Error (MSE) = $1/p \sum (y_n - \ddot{y}_n)^2$

Mean Absolute Percentage Error (MAPE) = $1/p \sum |y_n - \ddot{y}_n| / y_n$

Direction of change error = error when $y_n - y_{n-1} > 0$ and $\ddot{y}_n - y_{n-1} \leq 0$

or error when $y_n - y_{n-1} \leq 0$ and $\ddot{y}_n - y_{n-1} > 0$

where y_n is the actual forecast or the event that occurs, y_{n-1} is our most recent value before the forecast, $\ddot{y}_n$ is our prediction and p is the total number of points predicted (test size). The first measures MSE measures the accuracy of the forecast. The MAPE measure quantifies the relative accuracy of the prediction procedure and is calculated as a percentage than simple ratio. Another important measurement, the direction % success measures the ratio in percentage of the number of times the actual and predicted values move in the same direction (go up or down together) to the total number of predictions. The forecasting process is repeated with a given test data for states/patterns of size greater than two and a model with smallest k giving minimal error is selected. In our experiments k is iterated between $2 \leq k \leq 5$.

III. NOISE INJECTION

Gaussian noise was produced using C++ library. Gaussian noise array with standard deviation equal to 1 and mean equal to 0 is first produced with a seed *s* which ranges between 1 and 10. The size of the noise array equals the size of the time-series under consideration. The noise array $\tilde{N}[i]$ is first scaled between the [-1, +1] range. The amount of average noise added per pattern will differ for different seed *s*. In this paper, additive non-cumulative noise is used in the experimental section. For time series data value, y[*i*], it is contaminated as: y[*i*] = y[*i*](1 + δ.Ñ[*i*]). The constant δ defines the upper limit and has been set to 10% of a given data value, i.e. δ = .1. The noisy data is used for further analysis.

IV. TIME-SERIES DATA

In this paper, the analysis data has been selected from varied domains including physics, astrophysics and finance. Three of the benchmark series (A, D and E) considered here come from the Santa Fe competition. The fourth series is the real S&P index for US financial market (monthly data from August 1988 to August 1996). The details of these univariate series are introduced below:

Series A : This is a univariate time series measured in a Physics laboratory experiment. This data set was selected because it is an example of complicated behaviour in a clean, stationary, low-dimensional non-trivial physical system for which the underlying dynamic equations are well understood. There are a total of 1000 observations. The correlation between y_t and y_{t-1} for the original series is .53 and for the difference series is .27 (Figure 2 and 3)

Series D: This univariate time-series has been generated for the equation of motion of a dynamic particle. The series has been synthetically generated with relatively high-dimensional dynamics. There are a total of 4572 observations. The correlation between y_t and y_{t-1} for the original series is .95 and for the difference series is .72. (Figure 4 and 5)

Series E: This univariate time-series is a set of astrophysical data (variation in light intensity of a star). The data set was selected because the information is very noisy, discontinuous and non-linear. There are a total of 3550 observations. The correlation between y_t and y_{t-1} for the original series is .81 and for the difference series is -.44. (Figure 6 and 7)

Series S&P: This series represents the S&P index over a period of eight years from 1988 to 1996 (provided by the London Business School). This data is noisy and exponentially increasing in nature. There are a total of 2110 observations. The correlation between y_t and y_{t-1} for the original series is .99 and for the difference series is .04. (Figure 8 and 9)

**Figures 2-9 here*

The statistical characteristics of these series are summarised in Table 1.

**Table 1 here*

V. RESULTS

The performance capability of the Pattern Modelling and Recognition System is based on its ability to make accurate forecasts and correctly predict the direction of time-series change. The accuracy of the predictions is measured using the MSE and MAPE measures described earlier. The application of the technique should be preceded by the selection of appropriate parameters. In the context of PMRS, the optimal value of pattern size used for matching, i.e. k, must be selected through experimentation. The optimal k varies for different time-series. The selection of optimal k is based on comparing the performance of PMRS prediction with varying k. The model with the least complexity (minimal k) producing the minimal forecast error is selected (*law of parsimony*). In this section, the PMRS performance on noise-free time-series data is compared with the best performance obtained by adding Gaussian noise. These two results are compared together with a similar application of the statistical Exponential Smoothing method of forecasting. The next sub-section considers the change in forecast errors when varying the standard deviation of noise injected. Some important conclusions are drawn from the results presented.

PMRS performance on noise-free and noise-injected series

The difference between PMRS performance on noise-free and noise-injected time-series is shown in Table 2.

**Table 2 here*

Table 2 shows the performance comparison between PMRS application to noise-free and noise-injected series. First, the optimal value of pattern size k was determined as shown in Table 2. The time-series is divided into two parts: an estimation or training period, and a forecast or test period. The results are based on 90% data in the estimation period and 10% data in the test period. The number of forecasts therefore for different series are: series A (100 points), series D (457 points), series E (355 points) and S&P series (211 points). The first half of Table 2 summarises the MSE, MAPE and direction success % values when PMRS is applied to untreated series A, D, E and S&P. For the second half of Table 2, time-series data is contaminated by noise of varying seed *s* = 1 ... 10, and the PMRS performance is recorded (MSE(Ñ), MAPE(Ñ) and direction success %(Ñ)). The seed values are shown only for the best performance. We compare the results on noise-free and noise-injected data. The better performance is underlined. The performance can be compared across the rows (for different series). Here we observe that series A, E and S&P are more accurately predicted when time-series data is noise-injected. Their direction change is always more correctly predicted with noise-injected data, and either their MSE or MAPE is lower as desired compared to noise-free prediction. One important observation is the magnitude of change in performance. This is significantly pronounced for series A (Noise injected series direction prediction is improved by 5% and the new MSE is only 64% of the one with noise-free analysis). Significant difference in MAPE for the S&P series can also be noted. The PMRS predicted values of noise-injected series are plotted against the actual series values in Figures 10-13. Only a total of 100 predictions have been plotted. These are shown below:

Series A:

Series A exhibits a periodic behaviour. Figure 10 shows that the predictions have a very good match with the actual series behaviour. The error between actual and predicted values is evident on correctly predicting the amplitude of the series (the predicted values are higher on several occasions). However, there are no structural differences or phase lags between actual and predicted values.

**Figure 10 here*

Series D:

Series D is more difficult to predict than series A. Figure 11 however shows that PMRS predictions closely match the actual behaviour except in certain regions.

**Figure 11 here*

Series E:

Series E is one of the more difficult benchmarks to predict. The predictions follow the actual trend of the series very well. The predictions match the actual series magnitude very well but fail to follow the actual series direction in the short term.

**Figure 12 here*

Series S&P:

Figure 13 shows the prediction of the returns of the original S&P series. The difference series is forecasted because of the non-stationary nature of the original series. These predictions can be easily translated to the original S&P series forecasts. Figure 13 shows that PMRS predictions closely follow the actual observations when the series variance is low (in earlier observations). The predictions are less accurate towards the end of the series which shows a more chaotic behaviour with large variance.

**Figure 13 here*

PMRS and Exponential Smoothing

It is important to compare PMRS performance with statistical forecasting of series A, D, E and S&P. One popular method of forecasting time-series is the Exponential Smoothing (ES) method. This method follows the approach that future predictions are based on lagged values. The exponential smoothing method is a special class of ARIMA model (Delurgio, 1998) explained by the following equation:

$$y_{n+1} = \alpha(y_n + (1-\alpha)y_{n-1} + (1-\alpha)^2 y_{n-2} + (1-\alpha)^3 y_{n-3} + (1-\alpha)^4 y_{n-4} \ldots)$$

Exponential smoothing model is a good model for comparison with PMRS as depending on the value of α, it can act as different models. For large values of α nearing 1, it acts as a random walk model where y_{n+1} is the same as y_n. For smaller values of α, ES model is a linear combination of several previous lags and models those series that have noticeable auto-correlation for significant lags. By the using the optimal value of α, the model tries to minimise error over a number of predictions. In our experiments, the optimal values of the constant α for series A, D, E and S&P are .99, .99, .3 and .6 respectively. Table 3 shows the results of Exponential Smoothing forecasts on noise-free and noise-injected time-series. The better performances have been underlined.

**Table 3 here*

Some important conclusions should be drawn from Table 3; i) the difference between MSE, MAPE and % direction change on noise-free and noise-injected series is negligible for series A which is comparatively well behaved; ii) Noise-injected performance is superior for series D and S&P but inferior for series E; iii) The PMRS performance in Table 2 is superior for all series on almost all measures compared to Table 3; and iv) Noise injection has a pronounced effect on the PMRS and ES method of forecasting time-series data.

Effect of Noise Standard Deviation

It is important to study the variation in performance with respect to variable noise statistics, especially its standard deviation. In this paper, we study the change in the three error measures for a constant Gaussian noise mean and changing standard deviation from 0.3 to 3.0 as recommended by Jim et al. (1995). The results are shown in Appendix I. Some important conclusions are presented below:

- There is some correlation between the amount of average noise added and the MSE/MAPE error values for varying noise standard deviation for series A (Figures 14 has a similar trend to Figure 18 and Figure 22). This is not true for other series.
- There is similarity between the MSE and MAPE trends of the same series for data A, D and E.
- In series A and E, an increase in noise standard deviation leads to better MSE/MAPE performance (reduction in error exhibited by downward trend) before showing a slow upward trend (increase in error) for series A or a flat plateau for series E. In such series, the optimal noise variance can be determined by identifying a sharp minimum. The best predictions for change in series direction are also made at this sharp minimum value of the noise standard deviation (Figures 26 and 28).
- In series D, the MSE/MAPE performance is varied and alternating in nature. An increase in noise variance leads to several error minima. For best noise series selection, more exhaustive tests need to be performed to determine the least error model. The change in direction measure also has an alternating characteristic. There is however a lag difference between Figures 19 and 27, and 23 and 27. In other words, when the best MSE/MAPE value results, the direction success % measurement is not necessarily the best. The best combination for noise-series selection must be then selected on intuition and experience.
- Series S&P behaviour is interesting. An increase in noise variance leads to poor performances on the MSE measure (Figure 21). The change on MAPE is much smaller in comparison (Figure 25). In most financial domains, the direction of series change is a more important measure for practical purposes. This measure guides traders on when to stay in and out of the market for maximising their profits and minimising risks. The behaviour on this measure is most interesting and different than any other plots (Figure 29). An increase in noise first improves the performance before showing a saturation and slow decline. It seems that the best standard deviation value for predicting S&P series would be close to 1.

VI. CONCLUSION

Time-series behaviour prediction is an important activity in several domains. In certain domains such as finance, advanced computational techniques and expert human judgement combine to produce accurate forecasts. The proposed Pattern Modelling and Recognition System out-performs the statistical Exponential Smoothing method on predicting all four time-series considered in this paper. The PMRS algorithm is built on the existing computationally intelligent pattern recognition techniques. The hypothesis tested in this paper that the addition of controlled amounts of noise in time-series data improves prediction accuracy was found to be true. It is however important to bear in mind that the noise series characteristics are crucially important to the quality of results obtained. The addition of noise has a pronounced effect on the pattern matching ability of the PMRS algorithm. Further studies should be recommended which explore the role of noise in time-series analysis, both its addition and removal.

REFERENCES

[1] Azoff, M. E. *Neural Network Time Series Forecasting of Financial Markets*, John Wiley and Sons, 1994.

[2] Brillinger, D. R. *Time Series: Data Analysis and Theory*, McGraw-Hill, 1981.

[3] Burton, R. M. and Mpitsos, G. J., "Event-dependent control of noise enhances learning in neural networks," Neural Networks, vol. **5**, pp. 627-637, 1992.

[4] Chorafas, D. N. *Chaos Theory in the Financial Markets: Applying Fractals, FuzzyLogic, Genetic Algorithms, Swarn Simulation & the Monte Carlo Method to Manage Markets*, Probus Publishing Company, 1994.

[5] Delurgio, S. *Forecasting: Principles and Applications*, McGraw-Hill, 1998.

[6] Farmer, J. D. and Sidorowich, J. J. Predicting Chaotic Dynamics, in Dynamic Patterns in Complex Systems, J. A. S. Kelso, A. J. Mandell and M. F. Shlesinger (Eds.), pp. 265-292, Singapore: World Scientific, 1988.

[7] Jim, K., Horne, B. and Giles, C. L. "Effects of noise on convergence and generalisation in recurrent networks," *Neural Information Processing Systems* 7, G. Tesauro, D. Touretzky and T. Leen (eds.), MIT Press, p. 649, 1995.

[8] MacDonald, I. L. and Zucchini, W. *Hidden Markov and other models for discrete-valued time-series*, London:Chapman and Hall, 1997.

[9] Motnikar, B. S., Pisanski, T. and Cepar, D. Time-series forecasting by pattern imitation, OR Spektrum, 18(1), pp. 43-49, 1996.

[10] Murray A. F. and Edwards, P. J. "Synaptic weight noise during multilayer perceptron training: Fault tolerance and training improvements," *IEEE Transactions on Neural Networks*, vol. **4**, issue 4, pp. 722-725, 1993.

[11] Singh, S. "Effect of Noise on Generalisation in Massively Parallel Fuzzy Systems", Pattern Recognition, vol. 31, issue 11, pp. 25-33, 1998.

[12] Weigend**,** A. S. and Gersehnfield, N. A. 1994 *Time Series Prediction: Forecasting the Future and Understanding the Past*. eds. Reading, MA: Addison-Wesley.

Figure 1. Flowchart for the PMRS forecasting algorithm

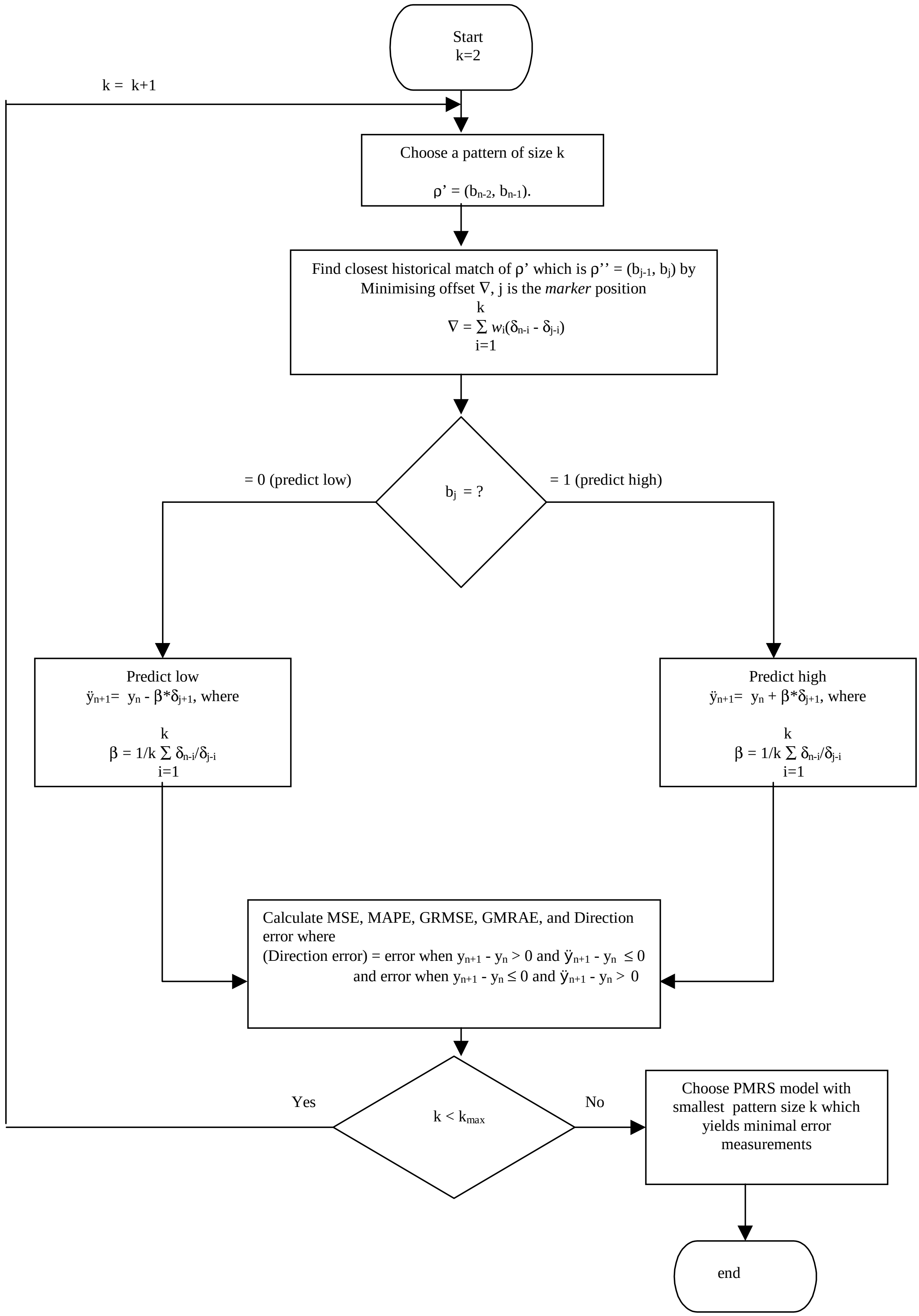


Figure 2. Plot of Series A

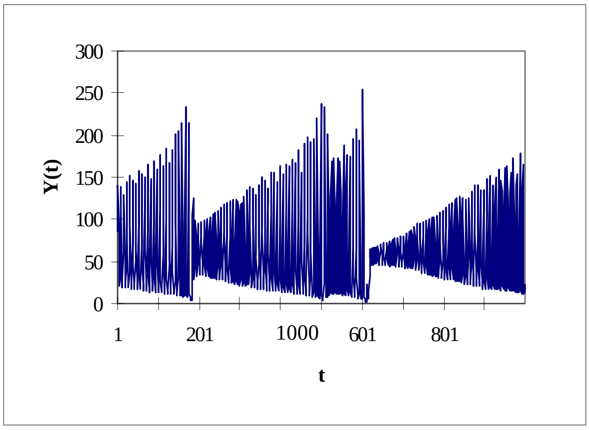


Figure 3. Plot of the difference of series A

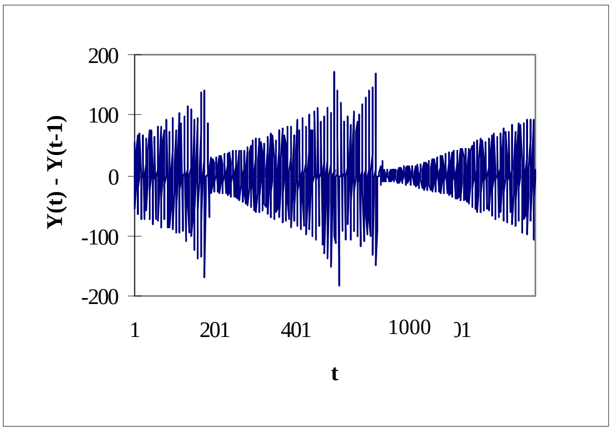


Figure 4. Plot of Series D

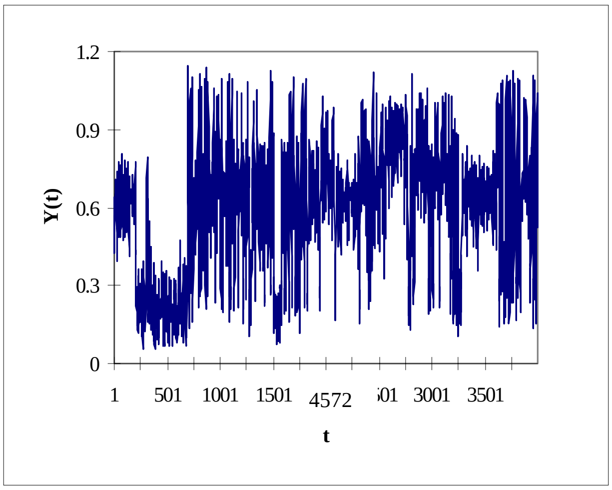


Figure 5. Plot of the difference of series D

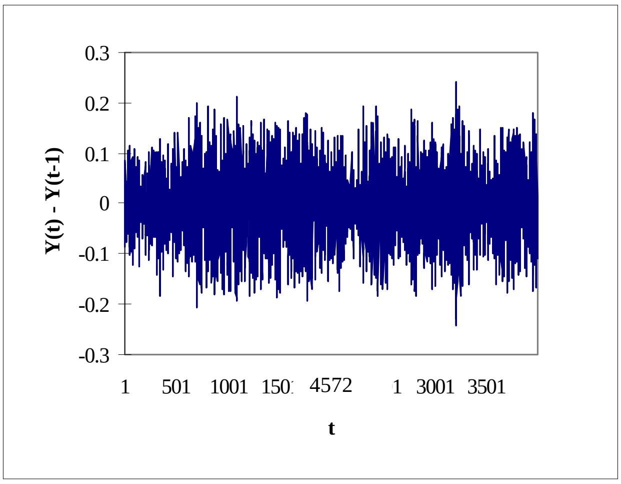


Figure 6. Plot of Series E

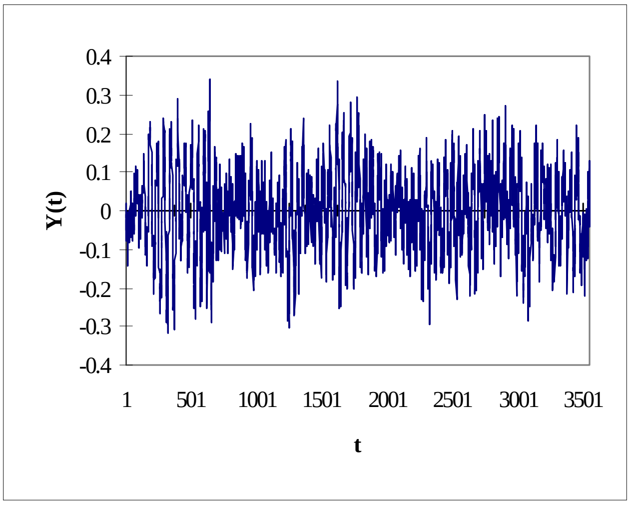


Figure 7. Plot of the difference of series E

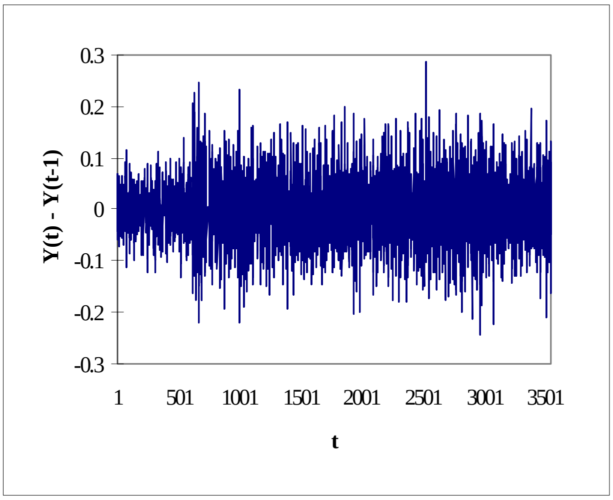


Figure 8. Plot of S&P index

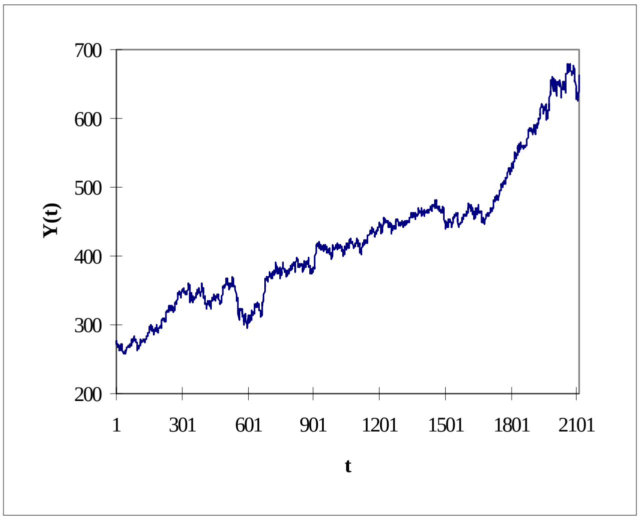


Figure 9. Plot of the difference of S&P index

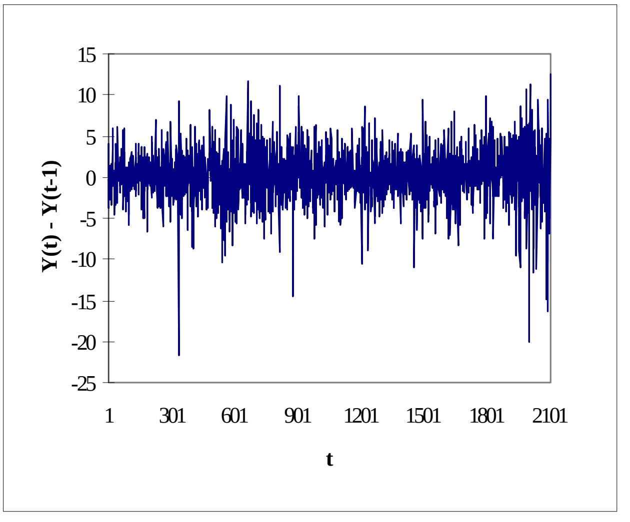


Figure 10. Actual plot of series A compared with the PMRS predictions on noise-injected series A

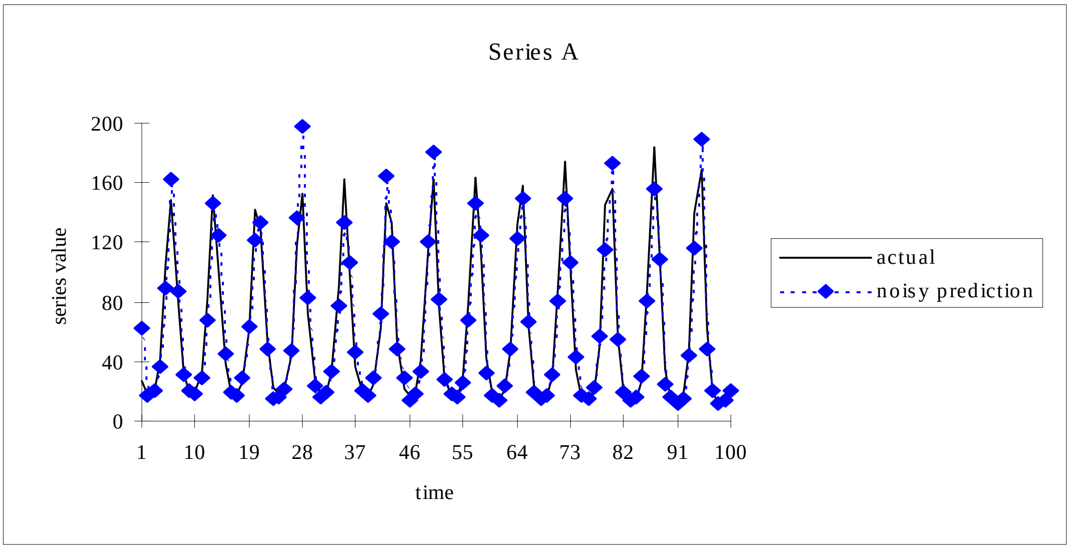


Figure 11. Actual plot of series D compared with the PMRS predictions on noise-injected series D

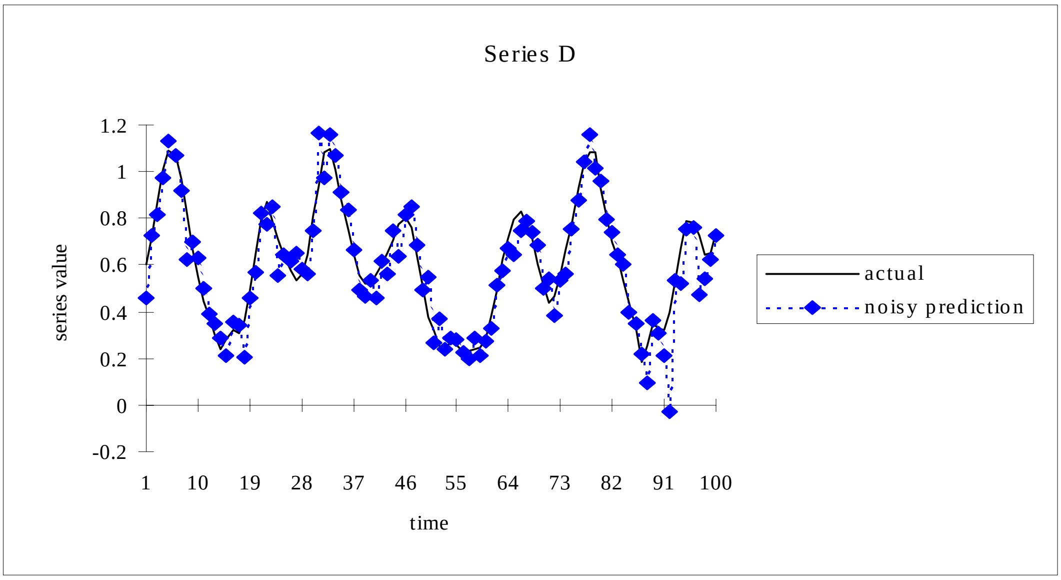


Figure 12. Actual plot of series E compared with the PMRS predictions on noise-injected series E

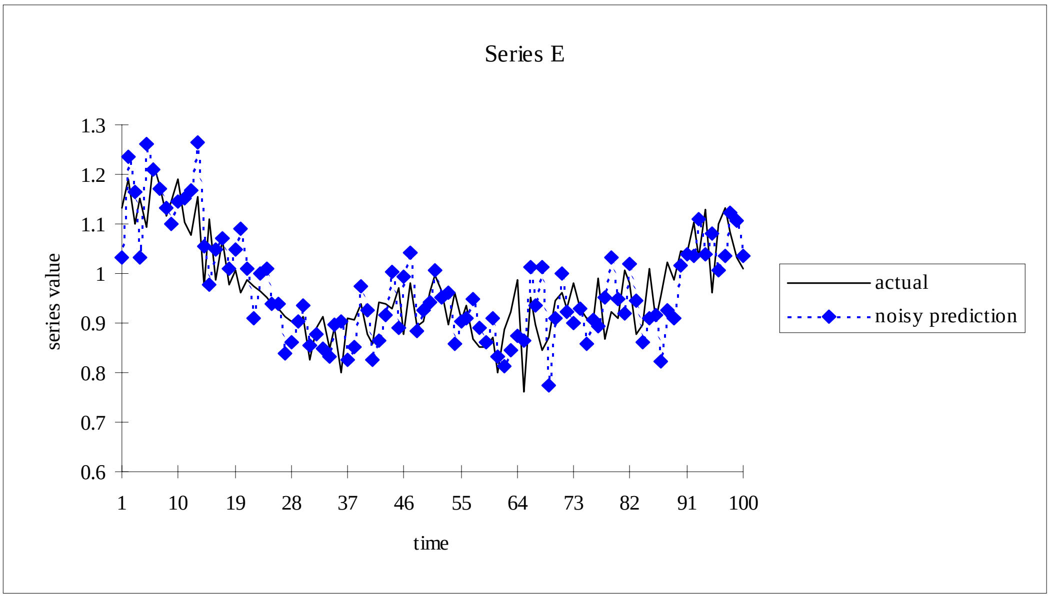
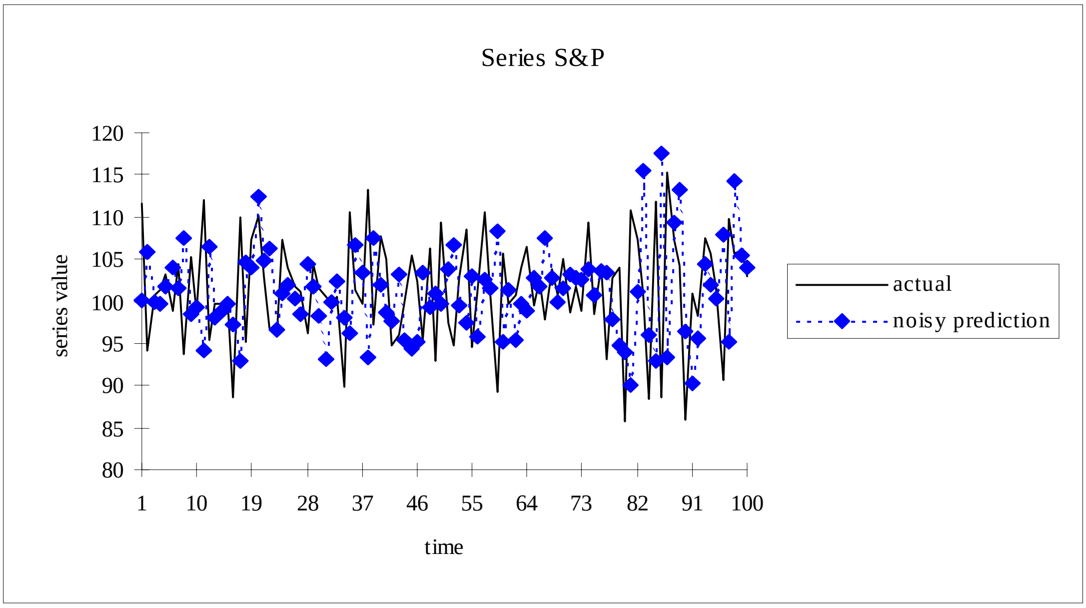


Figure 13. Actual plot of series S&P compared with the PMRS predictions on noise-injected series S&P



Appendix I (Figures 14-29)

Figure 14. Change in average noise per pattern for series A for varying noise standard deviation

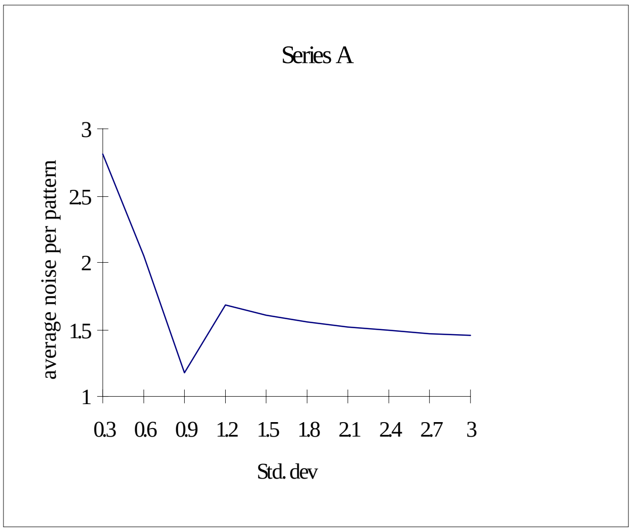


Figure 15. Change in average noise per pattern for series D for varying noise standard deviation

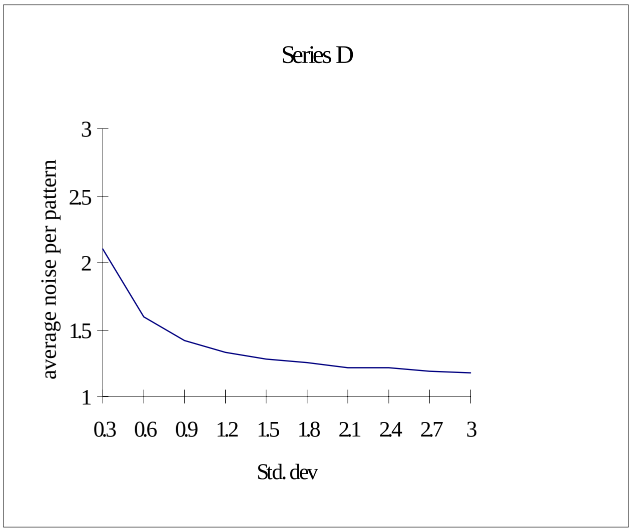


Figure 16. Change in average noise per pattern for series E for varying noise standard deviation

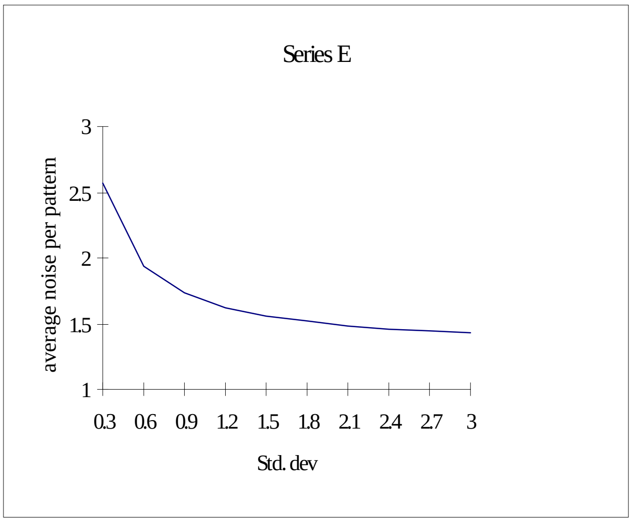


Figure 17. Change in average noise per pattern for series S&P for varying noise standard deviation

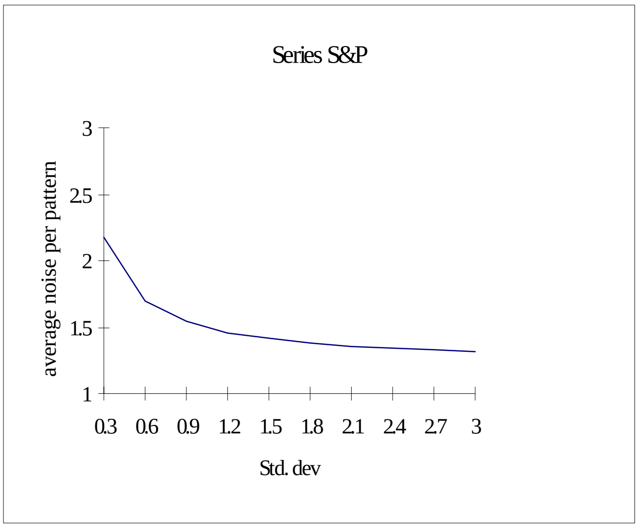


Figure 18. Change in MSE when predicting series A for varying noise standard deviation

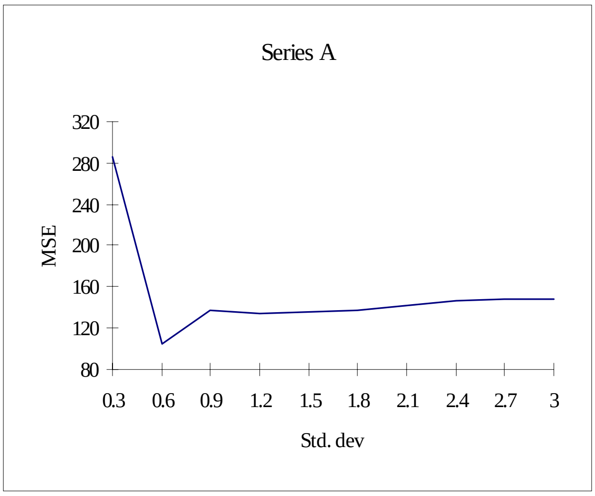


Figure 19. Change in MSE when predicting series D for varying noise standard deviation

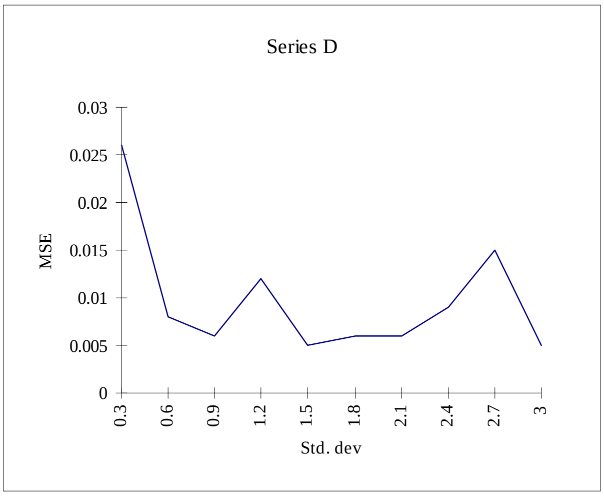


Figure 20. Change in MSE when predicting series E for varying noise standard deviation

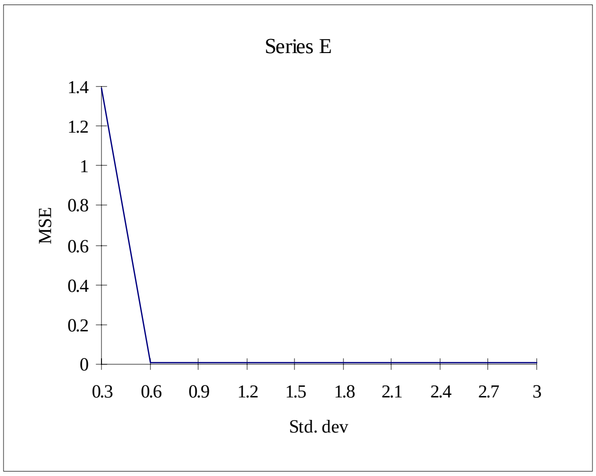


Figure 21. Change in MSE when predicting series S&P for varying noise standard deviation

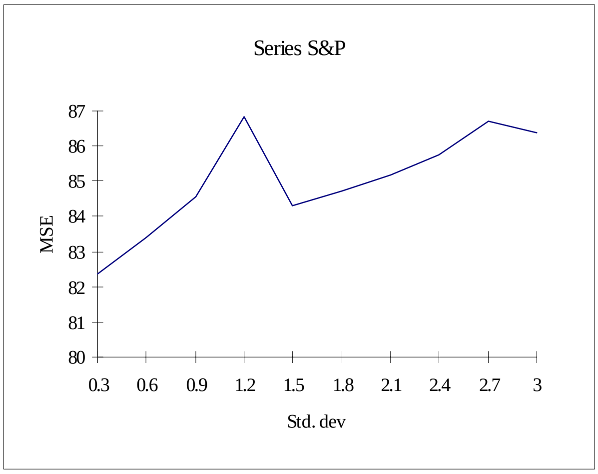


Figure 22. Change in MAPE when predicting series A for varying noise standard deviation

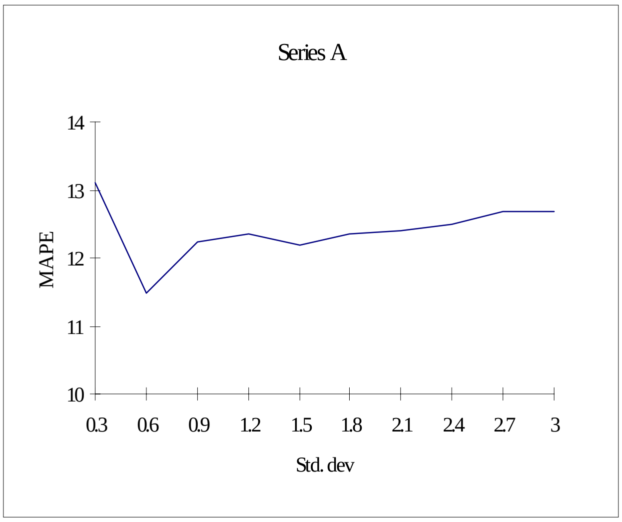


Figure 23. Change in MAPE when predicting series D for varying noise standard deviation

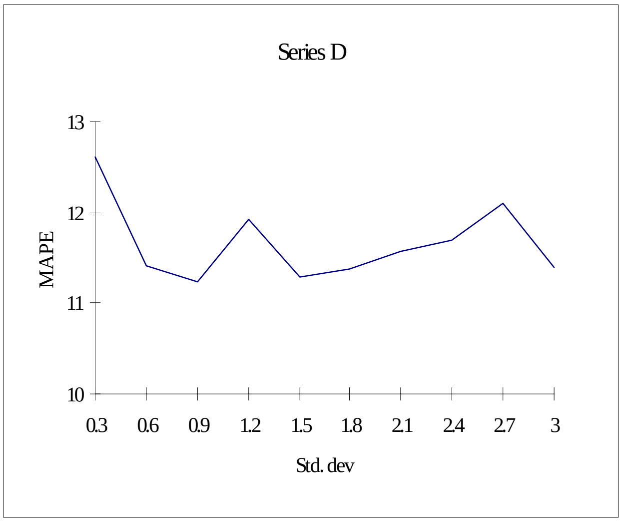


Figure 24. Change in MAPE when predicting series E for varying noise standard deviation

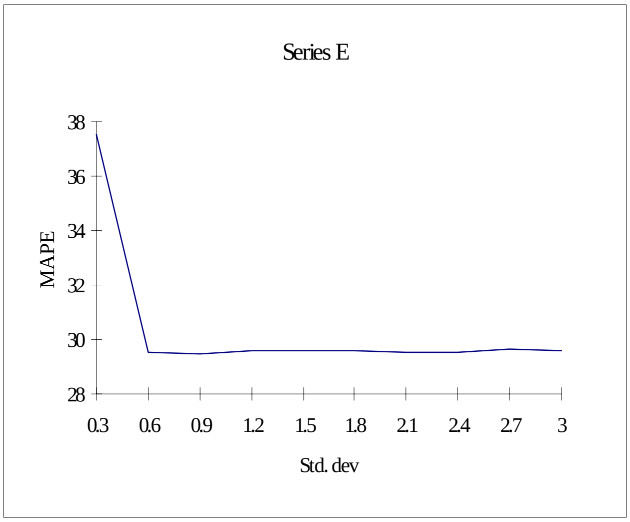


Figure 25. Change in MAPE when predicting series S&P for varying noise standard deviation

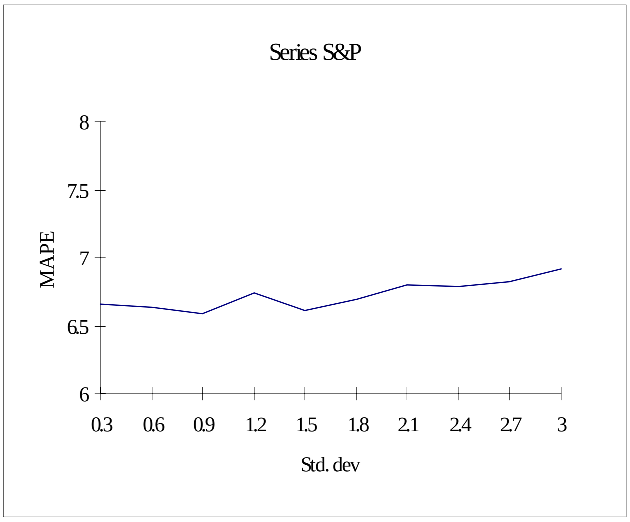


Figure 26. Change in direction success % when predicting series A for varying noise standard deviation

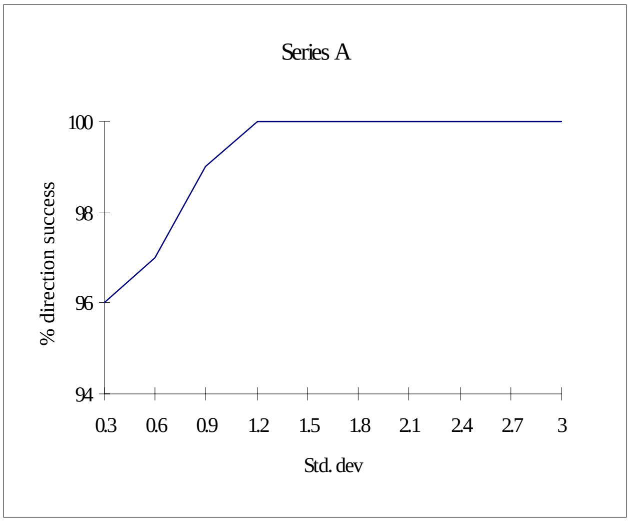


Figure 27. Change in direction success % when predicting series D for varying noise standard deviation

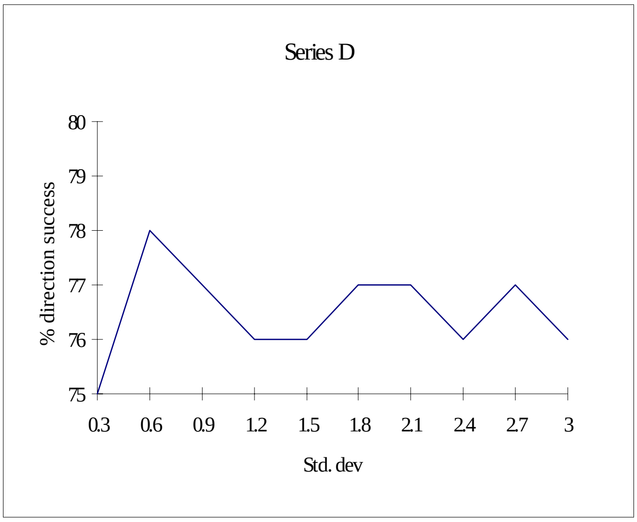


Figure 28. Change in direction success % when predicting series E for varying noise standard deviation

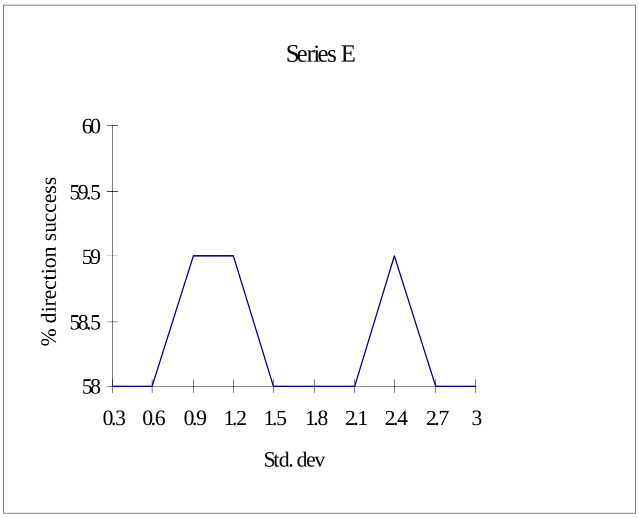


Figure 29. Change in direction success % when predicting series S&P for varying noise standard deviation

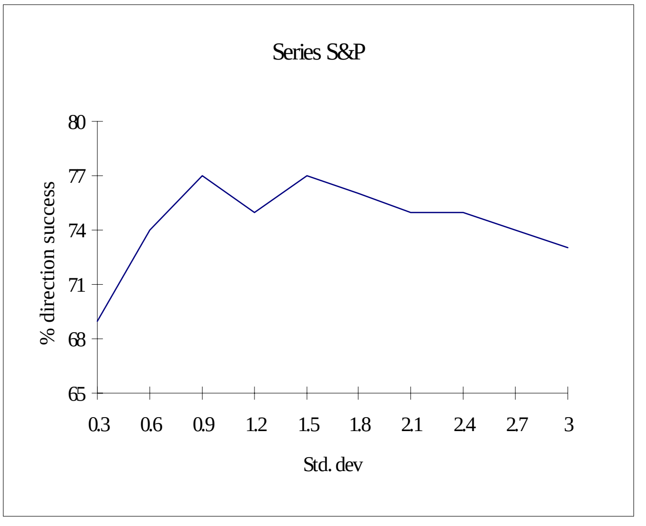


Table 1. Series A, D, E and S&P statistics

Series	Min	Max	Mean	SD	Size
A	2	255	59.90	46.87	1000
D	.05	1.15	.58	.23	4572
E	-.31	.34	0	.10	3550
S&P	257.10	678.50	422.74	100.32	2110

Table 2. Best performance comparison between noise-free and Gaussian noise-injected time-series forecasting using the Pattern Recognition and Modelling System

Series	k	MSE	MAPE	direction success %	MSE (Ñ)	MAPE (Ñ)	direction success % (Ñ)	Av. Noise per pattern
A	4	202.47	11.79	95	129.55	11.94	100	1.75 (s = 5)
D	3	.003	8.58	84	.005	10.99	77	1.38 (s = 4)
E	2	.009	31.36	56	.008	29.83	59	1.68 (s = 5)
SP	3	85.22	7.05	76	87.27	6.74	77	1.51 (s = 3)

Table 3. Best performance comparison between noise-free and Gaussian noise-injected time-series forecasting using the Exponential Smoothing method

Series	MSE	MAPE	direction success %	MSE (Ñ)	MAPE (Ñ)	direction success % (Ñ)	Av. Noise per pattern
A	<u>2346.31</u>	71.40	<u>27</u>	2373.91	<u>71.22</u>	27	1.58 (*s* = 9)
D	.12	20.55	<u>26</u>	<u>.007</u>	<u>13.60</u>	20	1.55 (*s* = 8)
E	<u>.011</u>	<u>30.07</u>	<u>52</u>	.032	36.51	50	1.37 (*s* = 1)
SP	104.64	7.73	<u>48</u>	<u>75.28</u>	<u>6.23</u>	47	1.31 (*s* = 9)