A LONG MEMORY
PATTERN MODELLING AND RECOGNITION SYSTEM
FOR FINANCIAL TIME-SERIES FORECASTING

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Singh, S. "A Long Memory Pattern Modelling and Recognition System for Financial Forecasting", Pattern Analysis and Applications, vol. 2, issue 3, pp. 264-273, 1999.

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ABSTRACT

In this paper, the concept of a long memory system for forecasting is developed. Pattern Modelling and Recognition Systems are introduced as local approximation tools for forecasting. Such systems are used for matching current state of the time-series with past states to make a forecast. In the past, this system has been successfully used for forecasting the Santa Fe competition data. In this paper, we forecast the financial indices of six different countries and compare the results with neural networks on five different error measures. The results show that pattern recognition based approaches in time-series forecasting are highly accurate and these are able to match the performance of advanced methods such as neural networks.

1. MOTIVATION

Time-series forecasting is an important research area in several domains. Traditionally, forecasting research and practice has been dominated by statistical methods. More recently, neural networks and other advanced methods on prediction have been used in financial domains [1-3]. As we get to know more about the dynamic nature of the financial markets, the weaknesses of traditional methods become apparent. In the last few years, research has focussed on understanding the nature of financial markets before applying methods of forecasting in domains including stock markets, financial indices, bonds, currencies and varying types of investments. Peters [4] notes that most financial markets are not Gaussian in nature and tend to have sharper peaks and fat tails, a phenomenon well know in practice. In the face of such evidence, a number of traditional methods based on Gaussian normality assumption have limitations making accurate forecasts.

One of the key observations explained by Peters [4] is the fact that most financial markets have a very long memory; what happens today affects the future forever. In other words, current data is correlated with all past data to varying degrees. This long memory component of the market can not be adequately explained by systems that work with short-memory parameters. Short-memory systems are characterised by using the use of last i series values for making the forecast in univariate analysis. For example most statistical methods and neural networks are given last i observations for predicting the actual at time $i+1$. For the sake of argument, such systems are termed as 'short memory systems' in this paper. Long memory systems on the other hand are characterised by their ability to remember events in the long history of time-series data and their ability to make decisions on the basis of such memories. Such a system is not constrained by how old the memory is. Hence, in such systems the forecasts are based on historical matches of market situations.

One strand of our motivation comes from the *Fractal Market Hypothesis*, Peters [4]. Peters [4] provided extensive results supporting his claim that markets have a fractal structure based on different investment horizons and that they cover more distance than the square root of time, i.e. they are not random in nature. Instead they consist of non-periodical cycles which are hard to detect and use in statistical or neural forecasting. He argues that financial markets are predictable but algorithms need long memories for accuracy in forecasting. The investment behaviour in markets has a repetitive nature over a long period of time and we believe that this can be detected using local approximation. Short memory systems can not capture the essence of this behaviour. This paper is also motivated by a study by Farmer and Sidorowich [5] who found that chaotic time-series prediction is several orders of magnitude better using local approximation techniques than universal approximators. Local approximation refers to the concept of breaking up the time domain into several small neighbourhood regions and analysing these separately. The well-known methods of local approximation for time-series prediction include chart representations, nearest neighbour methods [5] and pattern imitation techniques [6]. In this paper, we propose a new method of local approximation based on pattern matching that will be implemented in our Pattern Modelling and Recognition system.

A Pattern Modelling and Recognition System (PMRS) is a pattern recognition tool for forecasting. In this paper we approach the development of this tool with the aim of forecasting six different financial indices. The paper is organised as follows. We first discuss the concept of local approximation in forecasting and detail its implementation in the PMRS algorithm. The PMRS algorithm for forecasting is detailed next. The paper discusses the financial index data used in this study with their statistical characteristics. The results section compares the results obtained using the PMRS algorithm and backpropagation neural networks that are widely used in the financial industry. The results are compared on a total of five different error measures. Plots comparing PMRS and neural network predictions and actual series values for August 1996 are shown in Appendix I. The paper concludes by suggesting further areas of improvement.

2. LOCAL APPROXIMATION USING PMRS

The main emphasis of local approximation techniques is to model the current state of the time-series by matching its current state with past states. If we choose to represent a time-series as $\mathbf{y} = \{y_1, y_2, \ldots y_n\}$, then the current state of size one of the time-series is represented by its current value y_n. One simple method of prediction can be based on identifying the closest neighbour of y_n in the past data, say y_j, and predicting y_{n+1} on the basis of y_{j+1}. This approach can be modified by calculating an average prediction based on more than one nearest neighbours. The definition of the current state of a time-series can be extended to include more than one value, e.g. the current state s_c of size two may be defined as $\{y_{n-1}, y_n\}$. For such a current state, the prediction will depend on the past state s_p $\{y_{j-1}, y_j\}$ and next series value y^+_p given by y_{j+1}, provided that we establish that the state $\{y_{j-1}, y_j\}$ is the nearest neighbour of the state $\{y_{j-1}, y_j\}$ using some similarity measurement. In this paper, we also refer to *states* as *patterns*. In theory, we can have a current state of any size but in practice only matching current states of optimal size to past states of the same size yields accurate forecasts since too small or too large neighbourhoods do not generalise well. The optimal state size must be determined experimentally on the basis of achieving minimal errors on standard measures through an iterative procedure.

We can formalise the prediction procedure as follows:

$$\ddot{y} = \phi(s_c, s_p, y^+_p, k, c)$$

where $\ddot{y}$ is the prediction for the next time step, s_c is the current state, s_p is the nearest past state, y^+_p is the series value following past state s_p, k is the state size and *c* is the matching constraint. Here $\ddot{y}$ is a real value, s_c or s_p can be represented as a set of real values, k is a constant representing the number of values in each state, i.e. size of the set, and *c* is a constraint which is user defined for the matching process. We define *c* as the condition of matching operation that series direction change for each member in s_c and s_p is the same.

In order to illustrate the matching process for series prediction further, consider the time series as a vector $\mathbf{y} = \{y_1, y_2, \ldots y_n\}$ where n is the total number of points in the series. Often, we also represent such a series as a function of time, e.g. $y_n = y_t$, $y_{n-1} = y_{t-1}$, and so on. A segment in the series is defined as a difference vector $\boldsymbol{\delta} = (\delta_1, \delta_2, \ldots \delta_{n-1})$ where $\delta_i = y_{i+1} - y_i$, $\forall i$, $1 \le i \le n-1$. A pattern contains one or more segments and it can be visualised as a string of segments $\rho = (\delta_i, \delta_{i+1}, \ldots \delta_h)$ for given values of i and h, $1 \le i,h \le n-1$, provided that $h > i$. In order to define any pattern mathematically, we choose to tag the time series y with a vector of change in direction. For this purpose, a value y_i is tagged with a 0 if $y_{i+1} < y_i$, and as a 1 if $y_{i+1} \ge y_i$. Formally, a pattern in the time-series is represented as $\rho = (b_i, b_{i+1}, \ldots b_h)$ where b is a binary value.

The complete time-series is tagged as $(b_1, \ldots b_{n-1})$. For a total of k segments in a pattern, it is tagged with a string of k *b* values. For a pattern of size k, the total number of binary patterns (shapes) possible is 2^k. The technique of matching structural primitives is based on the premise that the past repeats itself. Farmer and Sidorowich [5] state that the dynamic behaviour of time-series can be efficiently predicted by using local approximation. For this purpose, a map between current states and the nearest neighbour past states can be generated for forecasting.

Pattern matching in the context of time-series forecasting refers to the process of matching current state of the time series with its past states. Consider the tagged time series (b_1, b_i, ... b_{n-1}). Suppose that we are at time *n* (y_n) trying to predict y_{n+1}. A pattern of size k is first formulated from the last k tag values in the series, ρ' = (b_{n-k}, ... b_{n-1}). The size k of the structural primitive (pattern) used for matching has a direct effect on the prediction accuracy. Thus the pattern size k must be optimised for obtaining the best results. For this k is increased in every trial by one unit till it reaches a predefined maximum allowed for the experiment and the error measures are noted; the value of k that gives the least error is finally selected. The aim of a pattern matching algorithm is to find the closest match of ρ' in the historical data (estimation period) and use this for predicting y_{n+1}. The magnitude and direction of prediction depend on the match found. The success in correctly predicting series depends directly on the pattern matching algorithm. The overall procedure is shown as a flowchart in Figure 1.

Figure 1 shows the implementation of the Pattern Modelling and Recognition System for forecasting. The first step is to select a state/pattern of minimal size (k=2). A nearest neighbour of this pattern is determined from historical data on the basis of smallest offset ∇. The nearest neighbour position in the past data is termed as "marker". There are three cases for prediction: either we predict high, we predict low, or we predict that the future value is the same as the current value. The prediction $\ddot{y}_{n+1}$ is scaled on the basis of the similarity of the match found. We use a number of widely applied error measures for estimating the accuracy of the forecast and selecting optimal k size for minimal error. The error measures are shown in Appendix III and will be discussed later. Another important measurement, the direction % success measures the ratio in percentage of the number of times the actual and predicted values move in the same direction (go up or down together) to the total number of predictions. The forecasting process is repeated with a given test data for states/patterns of size greater than two and a model with smallest k giving minimal error is selected. In our experiments k is iterated between $2 \leq k \leq 5$.

3. Financial Index Data

The financial indices used in this study include: German DAX, British FTSE, French FRCAC, SWISS, Dutch EOE and the US S&P series. All series data has been collected over a period of eight years between August 1988 and August 1996 (obtained by personal correspondence from London Business School). This data is noisy and exponentially increasing in nature. There are a total of 2111 observations in each series. The correlations between y_t and y_{t-1} for all series are .99. The statistical characteristics of the financial series are shown in Table 1.

Table 1 shows the auto-correlation for the first difference series (returns) within brackets ( ). Table 2 shows the correlation between returns of different financial indices. Some stock markets are closely related. This means that market movements in one country considerably affect the market movement in the closely related country. All correlation values above .5 have been highlighted. Since the actual indices show growth with time and they are non stationary, we perform all of our experiments with the difference series (returns).

4. ERROR MEASURES

The selection of appropriate error measures is important in identifying the differences in the forecasting ability of the PMRS and Neural Networks method. Fildes [7] notes that several statistical measures should be tried out before establishing the superiority of forecasting models. The most commonly used measure in forecasting is the root mean square measure (RMSE). This measure is very useful in measuring the accuracy of the forecasts but unfortunately it depends on the scale of the series, Armstrong and Collopy [8]. A more relative measure is the Mean Absolute Percentage error which shows the error in forecast relative to the actual value. Armstrong and Collopy [8] note that the RMSE measure is poor in reliability, fair in construct validity, poor in outlier protection, good in sensitivity and good in terms of its relationship to decisions. MAPE on the other hand is fair in reliability, good in construct validity, poor in outlier protection, good in sensitivity, and fair in terms of relationship to decisions. One main disadvantage of MAPE is that it puts a heavier penalty on forecasts that exceed the actual than on those that are less than the actual. Fildes [7] reports that another important measure is the geometric root mean square error (GRMSE) as it is well behaved and has a straightforward interpretation. Armstrong and Collopy [8] also recommend the use of geometric relative absolute error (GMRAE); this measure has a fair reliability, good construct validity, fair outlier protection, good sensitivity, and poor relationship to decisions. In general, absolute measurements are more sensitive and more relevant to decision making than relative measures. However, they tend to be poorer on outlier protection and reliability. In this paper we select the above four measures for comparing the PMRS and Neural Network results. We also use the % success in correctly predicting direction of series change measure which is widely used in the financial markets. We quote the RMSE, GRMSE, GMRAE measures on a per forecast basis. The formulae for calculating error measures are shown in Appendix III.

5. EXPERIMENTAL SECTION

In this section we first discuss the various issues involved in developing an optimal multi-layer perceptron architecture for forecasting. This forecasting model will be compared with PMRS.

Neural Networks

In the recent past, neural networks have been extensively used for forecasting stock markets [1-3, 11]. In practice, financial markets depend heavily on the standard neural network architectures and training algorithms; it will be realistic to assume that a multi-layer perceptron with backpropagation is a market standard. In this paper we use the standard MLP architecture for comparison on forecasting financial indices. The aim is to develop two separate neural networks for analysis: one for predicting the direction of index change (up or down movement), and another one for predicting the actuals. We first explain the input selection process, development of a neural network architecture, and training procedure.

Input/output selection

Gately [2] notes that several outputs may be selected for neural networks. We could predict the recent index change, the direction of series change, change in value over the forecast period, whether the trend is up or down, the rate of change, the quality of making a trade at a given time, volatility or change in volatility. In our analysis, we develop two separate neural networks: one for predicting the change in direction (up or down movement) and the other for predicting index returns. For the first network, the output is coded as a 1 if the series moves up, and a 0 if it goes down. For the second network, we first predict normalised returns directly. In this case the network output is a real number between the [0, 1] range.

The selection of inputs depends on what is being forecast (outputs). We use a linear procedure for determining the inputs to the two networks. We run a linear regression analysis for predicting the current return using the last ten returns, i.e. (y_{t-9}, … y_t). The relative importance of these inputs in correctly predicting the output is measured using t-statistic. The five most important variables are selected for further analysis. Here we make an assumption that the linear input selection is appropriate for a further non-linear analysis; this is in line with the work done by Refenes et al. [3]. Hence, for both networks there are the same five inputs. There is one output which in the first network is a 0 or 1 to predict whether the series goes up or down, and a real number for the second network to predict the actual changes. The number of hidden nodes are selected using the procedure discussed below.

Network architecture/hidden nodes

We follow the guidelines set by Weiss and Kulikowski [12]. The aim is to select a model of minimal complexity which leads to minimal generalisation error. We increment the number of hidden nodes in a stepwise manner to achieve the optimal configuration. Table 3 shows the final architectural and training details of neural networks used for predicting direction and series value of index returns. Each neural network consists of an input layer, hidden layer and output layer.

Training procedure

The training procedure consists of dividing the data into two parts: estimation period data and test period data. The estimation period data is used in training. We use 90% of the total data (total data is 2111 observations in all indices) in the estimation period and the remaining 10% in the test set. The training/test procedure is performed using the Stuttgart Neural Network Simulator using the standard backpropagation algorithm.

6. RESULTS

The first result comes from PMRS predicting the direction of series change; positive or negative relative to the current position. This is one of the most important measures in financial markets. The results for the different financial indices are shown in Table 4. For all indices, PMRS and Neural Network models are able to predict nearly three out of four times the correct direction of move. In Table 4 shows that the PMRS algorithm is very good at predicting the direction of the series change. This confirms that the match between the current market situation and the historical market situation is a crucial link for making accurate forecasts. The PMRS and NN performances are equally good on FRCAC and S&P index. The neural network is better on DAX and FTSE index whereas PMRS is better on SWISS and EOE. Traders in the financial markets are very much interested in forecasting the direction of index change. At a given time, the value of the index is known. The knowledge whether the financial index will be higher or lower the next day is important for the trading buy-sell-hold strategy. If the index value will be lower the next day, it may be useful to sell stocks, if it will be higher then it is better to hold on to your stocks and buy some now. A number of international companies that deal with more than one currency for business are also heavily dependent on the market changes. The ability to correctly forecast the direction of index change translates easily into the exchange rates of currencies and companies can save monies with large transactions through accurate predictions, Levich [13]. We next compare the RMSE, MAPE, GRMSE and GMRAE error measures of the two methods in Table 5.

In Table 5, the PMRS algorithm and Neural Networks perform significantly well. We compare the best PMRS and neural network models. The arithmetic Root Mean Square Error (RMSE), Geometric Root Mean Square Error, and Mean

Absolute Percentage error is lower for neural networks. Neural networks also have a slight edge over PMRS when compared to a random walk model shown by the GMRAE measure. The error measure GMRAE is important in measuring how well both models perform compared to the random walk model. The best method on this measure will generate a value of 0, the worst value will be ∞, and when the proposed method performs as good as the random walk model, then GMRAE should be equal to 1. Hence, the lower the value on this measure, relative to 1, the better the proposed method is compared to random walk. Since RAE is a ratio, in order to calculate how many times better than random walk, we can take an inverse of the value generated on this measure. For example, on DAX prediction, the difference between a random walk output and the actual series value is 90 times more than the difference between PMRS prediction and actual series value, and 125 times more for neural networks. The results are similar for other indices.

The key finding is that PMRS performs very efficiently and is quite close to neural network performance on several measures. These results coupled with Table 4 show that the PMRS algorithm is both accurate in forecasting the direction and magnitude of change. Without a doubt, PMRS approach offers the promise of better performance than neural networks with further modification.

An important step towards establishing PMRS as a good forecasting model is to show that the errors in prediction are not biased (low auto-correlation between error at different lags). Table 6 shows the auto-correlation between the PMRS residuals at different lags. Fortunately, the auto-correlations are low proving that the forecasts are not biased in either direction. We also use a *t*-test to confirm whether the mean error is statistically significantly different from zero, Delurgio[9]. The simple test is:

Null hypothesis: $\hat{e} = 0$; if this hypothesis is proven wrong, then accept

Alternative hypothesis: $\hat{e} < 0$ or $\hat{e} > 0$

A calculated t value is compared to the *t*-value from the *t*-table. If the calculated value is less than or equal to the table value then we accept the null hypothesis, and otherwise we reject it. The *t* value from the table for 95% confidence for *n*=210 samples (10% of total samples are predicted) is equal to 1.96. The *t* value is calculated by:

$$t\text{-calculated} = \hat{e} / (S_e/\sqrt{n})$$

where $\hat{e}$ is the mean error, S_e is standard deviation of errors about the mean $\hat{e}$ and *n* is the total number of samples. Table 7 shows the results for all indices. In each case, we accept the null hypothesis leading us to conclude that there is no bias and 95 percent of the errors lie within 1.96 standard errors of the mean value of zero.

The PMRS algorithm can be improved on these measures by following these guidelines:

- The PMRS algorithm could be optimised for the weights w_i as shown in Figure 1. In this paper, we have used all $w_i = 1$.
- The PMRS algorithm could be capped for a maximum and a minimum forecast avoiding wild estimates.
- The PMRS algorithm can select more than one nearest neighbour segments when matching and take an average of forecasts based on them. This technique may also be tried using extrapolation.
- One important advantage of PMRS is their real-time functioning. On a Pentium 200 MHz processors, it takes less than 1 second per forecast.

We next take one month August 1996 data of the different financial indices and plot both PMRS and NN predictions against the actuals. We only plot one month's data since larger plots do not exhibit in sufficient detail the differences between the actual and predicted values. Figures 2 to 7 in Appendix I show the plots of the different time-series predictions. As it can be seen, the PMRS forecasts are accurate in following the trend of the time series and the daily changes. This argument is further supported by the scatter plots shown in Appendix II. The plots show strong positive correlation between actual series values and PMRS predictions.

7. CONCLUSION

In this paper we have shown that a pattern recognition algorithm for forecasting is an alternative solution to using statistical methods that have limitations in non-linear domains, or neural networks that have long training times and require substantial experience in their prior use. The PMRS performance was found to be in close competition with neural networks; it is expected that further research on these predictors will yield superior results in the future. The PMRS tool is attractive for several reasons: i) pattern matching tasks approximate local neighbourhoods; ii) such tools require lesser number of parameters that need optimisation compared to sophisticated recurrent and backpropagation networks; and iii) PMRS works in real-time because of its non-iterative nature unlike neural networks, and are particularly suited in stock markets for predicting tick-type data. A short memory system uses inputs (y_{t-h}, … y_t) for predicting y_t and it has a memory equal to h. However, for a long memory system such as PMRS, the memory h for different test patterns is dynamic and is determined by the matching process. One important advantage of PMRS is that the historical memory triggered teaches us something about the time-series (trading behaviour). The distance between the current time, and the historical time when memory was triggered, i.e. value of h, showing the closest match can be statistically studied. Table 8 shows the memory characteristics of PMRS showing an average memory of more than 1000 days. It is interesting to note that the average memory across different markets is very similar and quite large. These results are in agreement with Peters [4] which confirm that financial markets have long memories of approximately four years. The minimum and maximum values in the table show how extreme these memories can become. Finally, the standard deviation shows how varied these memories are across different test patterns; the standard deviations are very large which emphasises that a dynamic rather than fixed model for prediction is a more natural way of forecasting financial series. Some of the work on using pattern recognition techniques for stock market forecasting has been recently implemented in the Norwegian stock market with encouraging results [14]. We expect that PMRS technology will be further developed and fine-tuned for practical trials.

REFERENCES

[1] M. E. Azoff, *Neural Network Time Series Forecasting of Financial Markets*, John Wiley, 1994.

[2] E. Gately, *Neural Networks for Financial Forecasting*, John Wiley, 1996.

[3] A. N. Refenes, N. Burgess and Y. Bentz, "Neural networks in financial engineering: A study in methodology," *IEEE Transactions on Neural Networks*, vol. 8, no. 6, pp. 1222-1267, 1997.

[4] E. Peters, *Fractal Market Hypothesis: Applying Chaos Theory to Investment and Economics,* John Wiley, 1994.

[5] J. D. Farmer, and J. J. Sidorowich, Predicting chaotic dynamics, in *Dynamic patterns in complex systems*, J. A. S. Kelso, A. J. Mandell and M. F. Shlesinger (Eds.), pp. 265-292, Singapore: World Scientific, 1988.

[6] B. S. Motnikar, T. Pisanski and D. Cepar "Time-series forecasting by pattern imitation", *OR Spektrum*, 18(1), pp. 43-49, 1996.

[7] R. Fildes, "The evaluation of extrapolative forecasting methods", *International Journal of Forecasting*, vol. 8, pp. 81-98, 1992.

[8] J. S. Armstrong, J. S. and F. Collopy, "Error measures for generalising about forecasting methods: Empirical comparisons", *International Journal of Forecasting*, vol. 8, pp. 69-80, 1992.

[9] S. Delurgio, *Forecasting: Principles and applications*, McGraw-Hill, 1988.

[10] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, J. Newton, E. Parzen and R. Winkler, *The forecasting accuracy of major time-series methods*, Chichester: John Wiley, 1984.

[11] J. Kingdon, Intelligent Systems for Financial Engineering, Springer, 1997.

[12] S. M. Weiss, and C. A. Kulikowski, *Computer systems that learn*, Morgan Kaufmann, San Mateo, CA, 1991.

[13] Levich, R. M., *International Financial Markets: Prices and Policies*, McGraw-Hill, 1987.

[14] Linlokken, G., "Syntactical pattern recognition in financial time-series," Proceedings of the Norwegian Image Processing and Pattern Recognition Society Conference, Oslo, 1996.

Figure 1. Flowchart for the PMRS forecasting algorithm

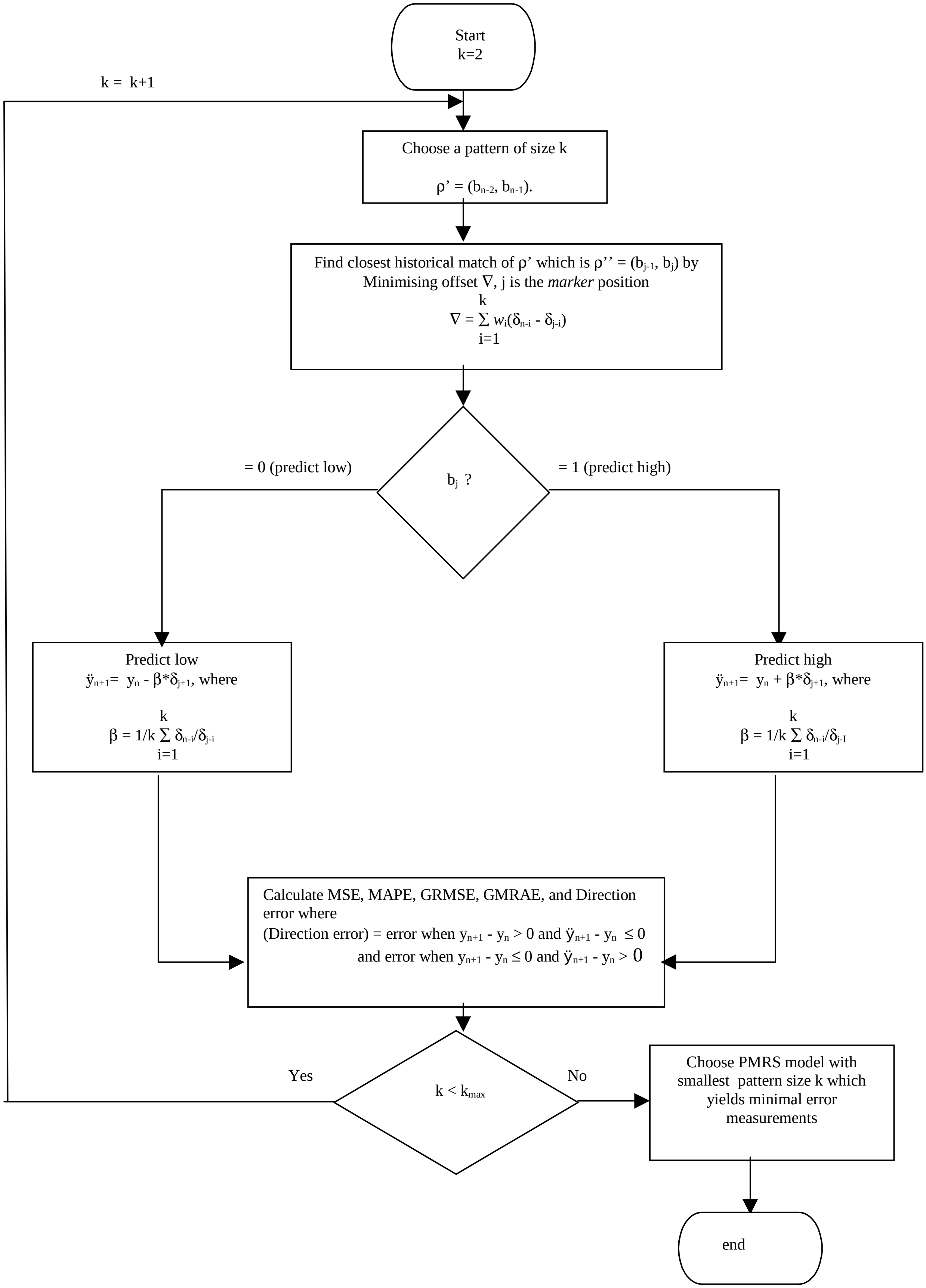


Table 1. Financial series statistics

Series	Min	Max	Mean	SD	Size	Autocorrelation $r(y_t, y_{t-1})$
DAX	1152	2583	1798	347	2111	.99 (-.50)
FTSE	1730	3857	2718	550	2111	.99 (-.48)
FRCAC	1276	2355	1862	201	2111	.99 (-.47)
SWISS	1287	3810	2165	635	2111	.99 (-.46)
EOE	729	1458	1030	187	2111	.99 (-.47)
S&P	257	678	422	100	2111	.99 (-.50)

Table 2. Correlation between changes in different financial indices

Series	DAX	FTSE	FRCAC	SWISS	EOE	S&P
DAX	1.00	.43	**.57**	**.61**	**.63**	.18
FTSE	.43	1.00	**.60**	**.50**	**.66**	.32
FRCAC	**.57**	**.60**	1.00	**.56**	**.70**	.25
SWISS	**.61**	**.50**	**.56**	1.00	**.66**	.22
EOE	**.63**	**.66**	**.70**	**.66**	1.00	.25
S&P	.18	.32	.25	.22	.25	1.00

Table 3. The optimal neural network architecture for predicting financial indices

Series	Direction Net Hidden Nodes	Direction Net Learning rate	Actual Net Hidden Nodes	Actual Net Learning rate
DAX	10	.5	10	.005
FTSE	5	.2	20	.005
FRCAC	15	.5	25	.005
SWISS	15	.5	15	.005
EOE	20	.5	20	.005
S&P	10	.5	15	.005

Table 4. The direction success rate % for the predicting different financial indices using PMRS with varying segment size k and Neural Networks (forecasting 10% of total data)

Index	Optimal k	PMRS	NN	% size predicted
DAX	5	73	77	10
FTSE	5	71	75	10
FRCAC	3	72	72	10
SWISS	3	72	67	10
EOE	3	75	74	10
S&P	3	76	76	10

Table 5. The best performances of PMRS and Neural Networks forecasting 10% of total data

Series / Method	Segments K	RMSE	MAPE	GRMSE	GMRAE
DAX					
PMRS	5	3.1	1.5	1.8	.011
NN		2.0	1.4	1.1	.008
FTSE					
PMRS	5	3.4	1.1	1.8	.010
NN		2.6	1.3	1.4	.008
FRCAC					
PMRS	3	2.6	1.5	1.6	.008
NN		1.4	1.3	1.0	.004
SWISS					
PMRS	2	4.8	1.5	2.5	.011
NN		2.3	1.2	1.4	.003
EOE					
PMRS	3	1.3	1.1	0.7	.008
NN		1.0	1.3	0.6	.007
S&P					
PMRS	3	0.6	1.1	0.3	.008
NN		0.4	1.1	0.2	.004

Table 6. Auto-correlation *r* between residuals at time lags 1 to 5 for forecasting 10% of total data using PMRS

Series	$r(e_t, e_{t-1})$	$r(e_t, e_{t-2})$	$r(e_t, e_{t-3})$	$r(e_t, e_{t-4})$	$r(e_t, e_{t-4})$
DAX	-.65	.17	-.08	.14	-.13
FTSE	-.68	.25	-.14	.16	-.14
FRCAC	-.65	.17	-.04	.07	-.06
SWISS	-.57	.27	-.21	.19	-.10
EOE	-.63	.17	-.11	.19	-.17
S&P	-.50	.12	-.09	.08	-.08

Table 7. Results on t-test for establishing that the errors are normally distributed for forecasting 10% of total data with PMRS

Index	t-Calculated	t-Table	Bias
DAX	.26	1.96	No
FTSE	.21	1.96	No
FRCAC	.30	1.96	No
SWISS	.34	1.96	No
EOE	.08	1.96	No
S&P	.25	1.96	No

Table 8. Memory statistics for the PMRS algorithm on forecasting 10% of financial index data

Series	Min	Mean	Max	Std. Dev.
DAX	24	1007	2040	583
FTSE	89	1117	2029	508
FRCAC	54	1019	2062	553
SWISS	47	884	1998	523
EOE	51	789	2000	458
S&P	35	1126	2051	505

Appendix I

Figure 2. DAX forecasts and actual index value for August 1996

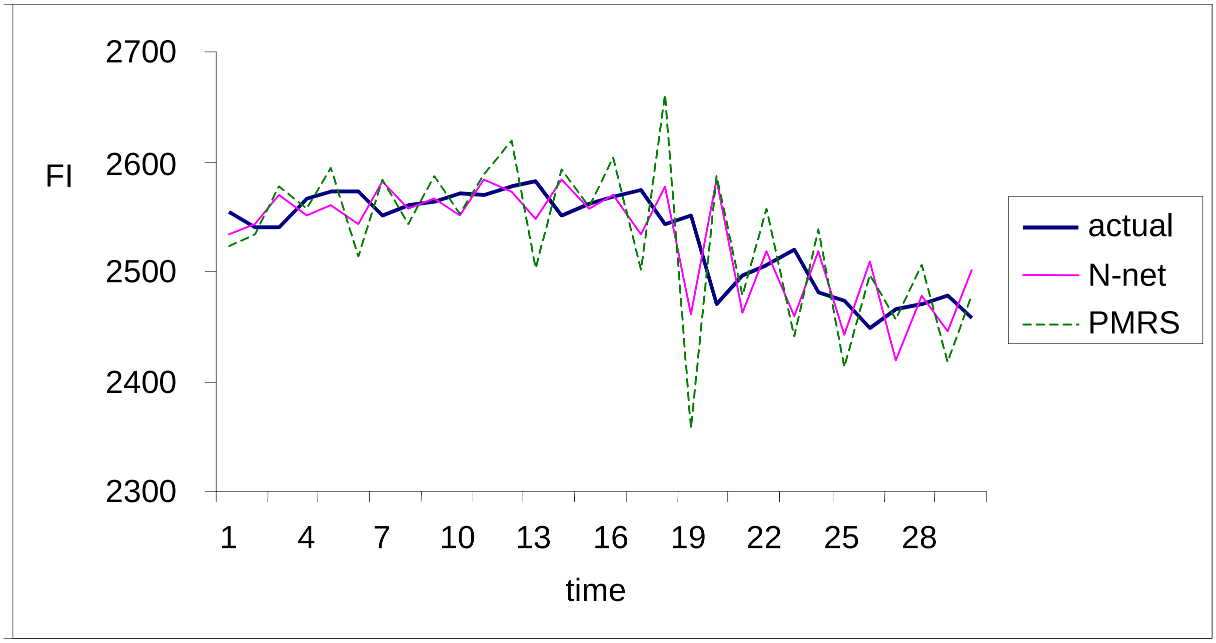


Figure 3. FTSE forecasts and actual index value for August 1996

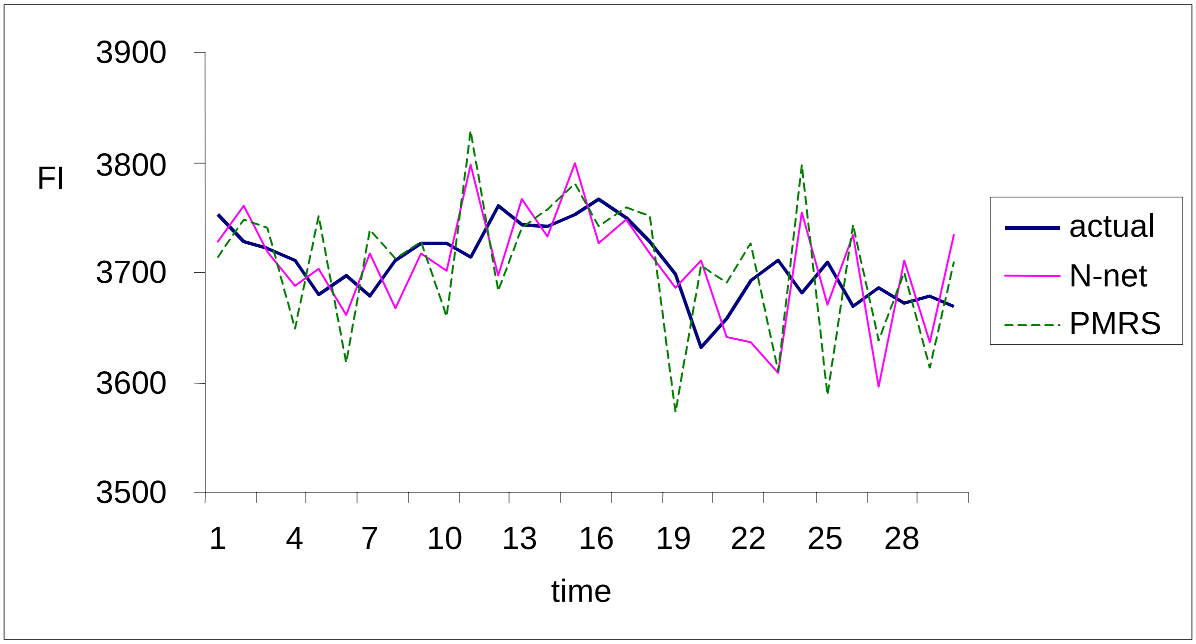


Figure 4. FRCAC forecasts and actual index value for August 1996

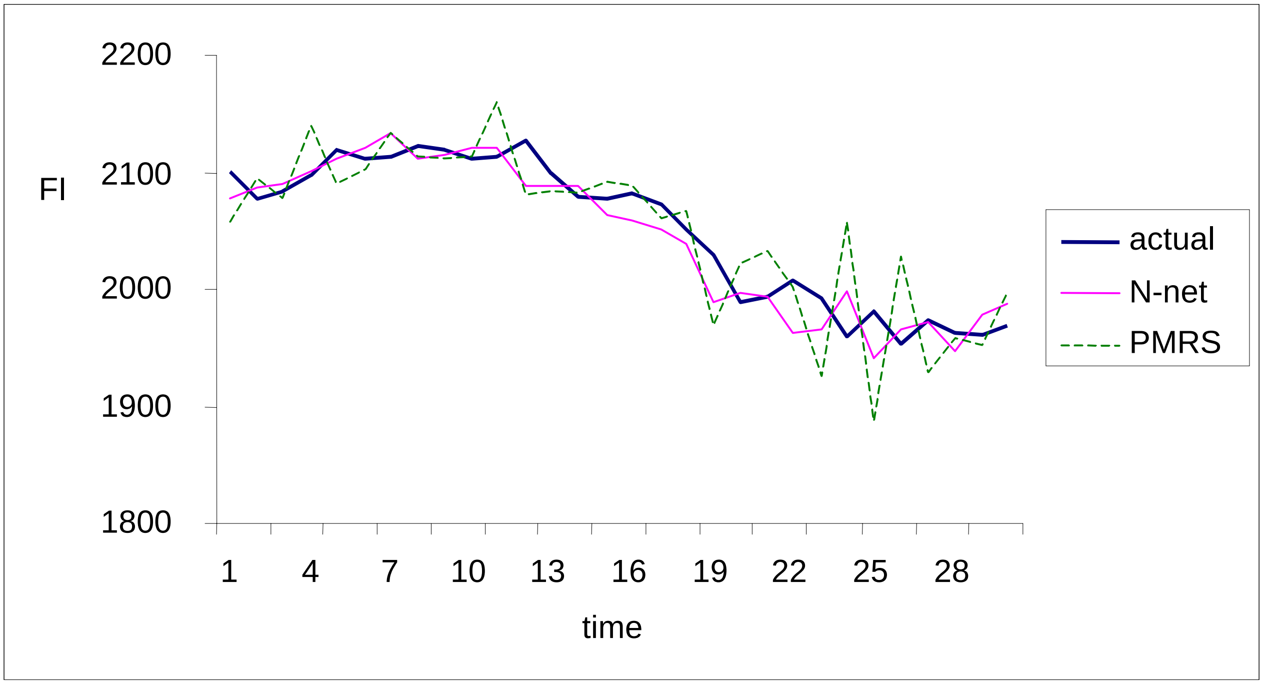


Figure 5. SWISS forecasts and actual index value for August 1996

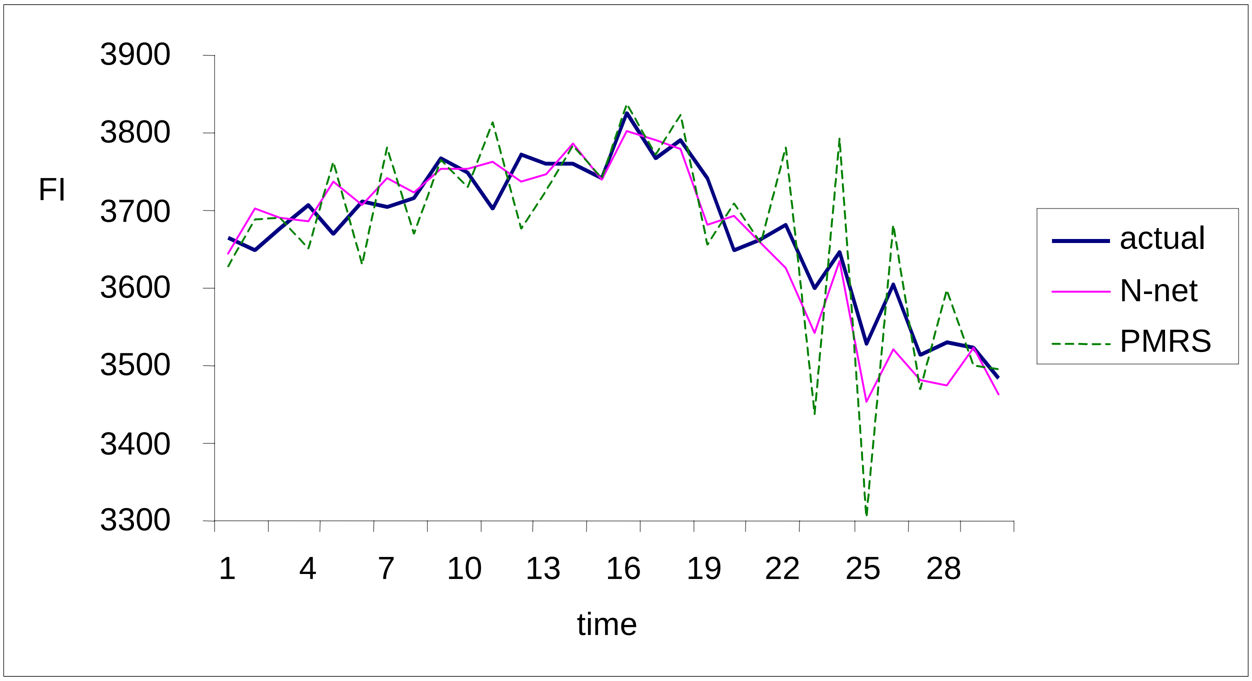


Figure 6. EOE forecasts and actual index value for August 1996

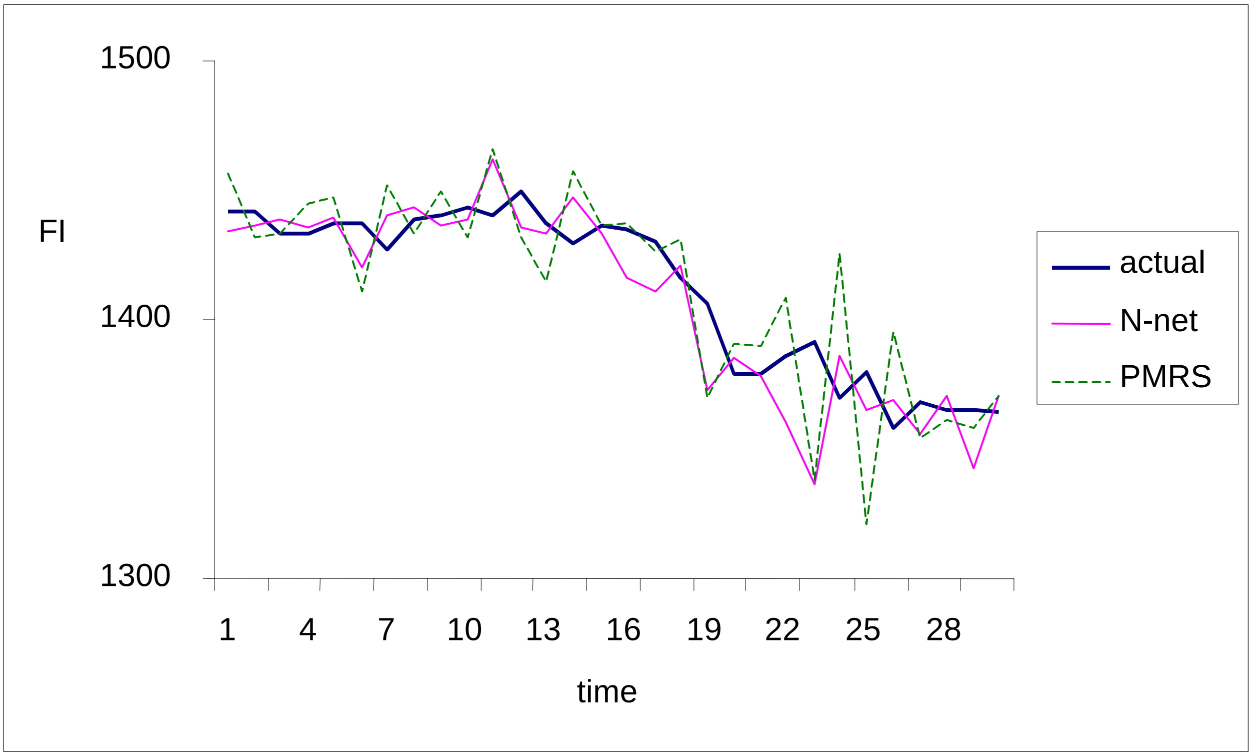
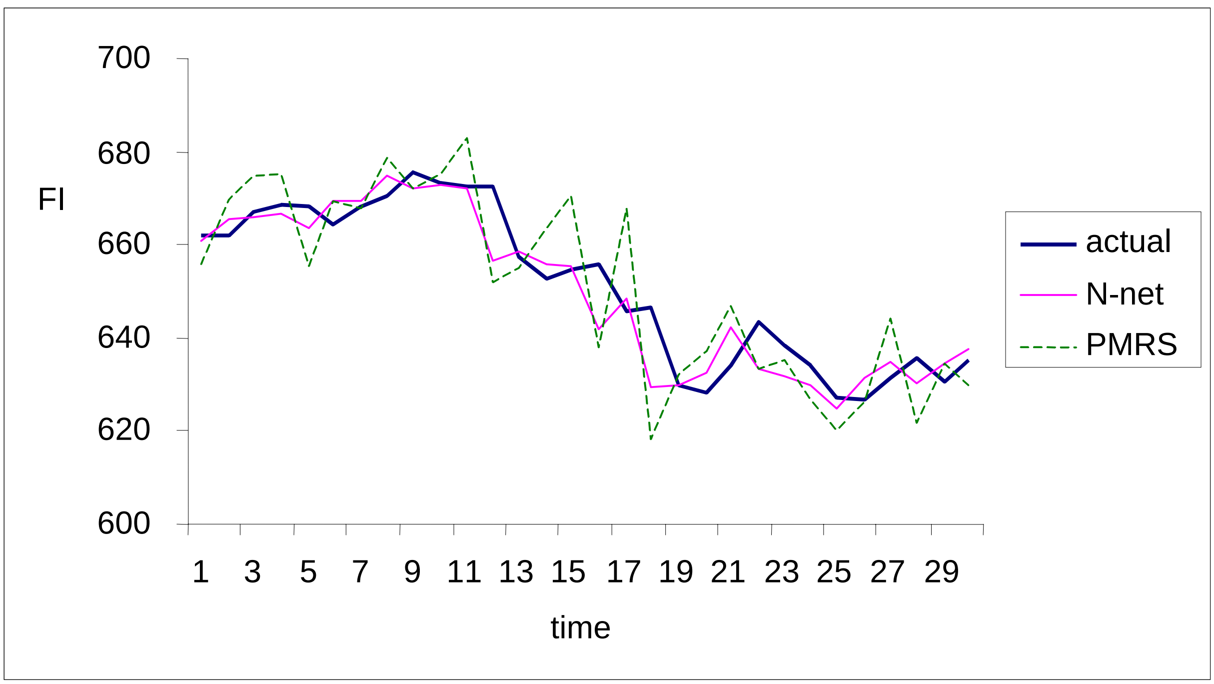
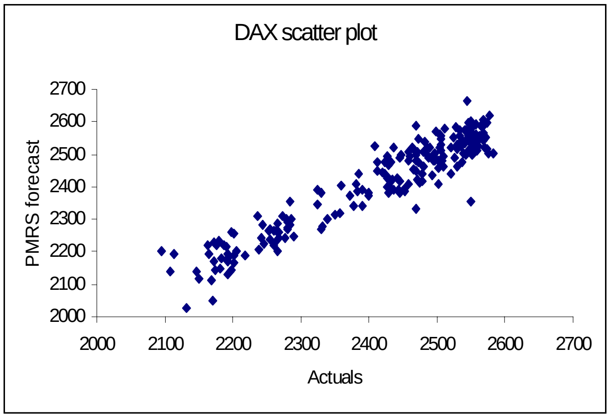
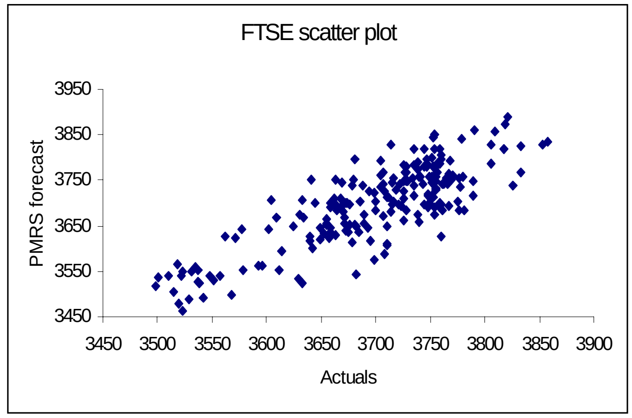


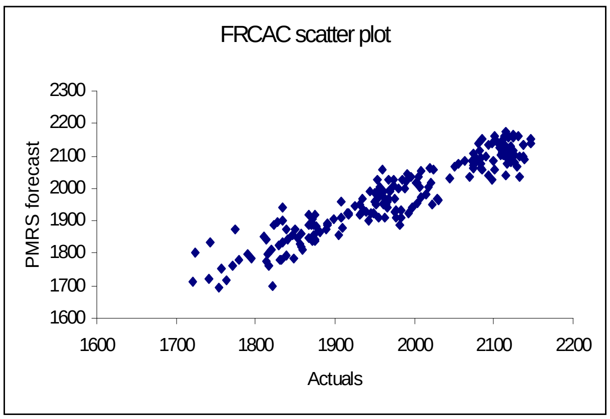
Figure 7. S&P forecasts and actual index value for August 1996

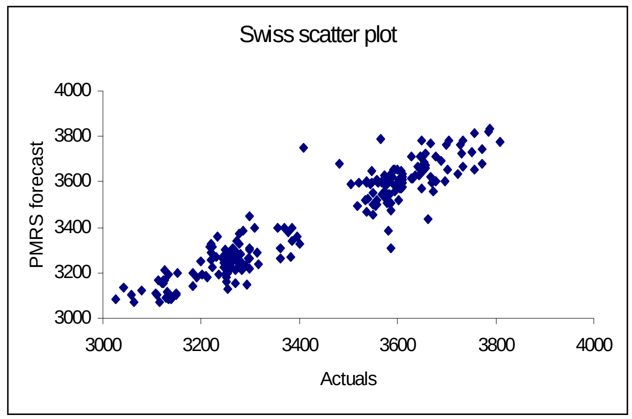


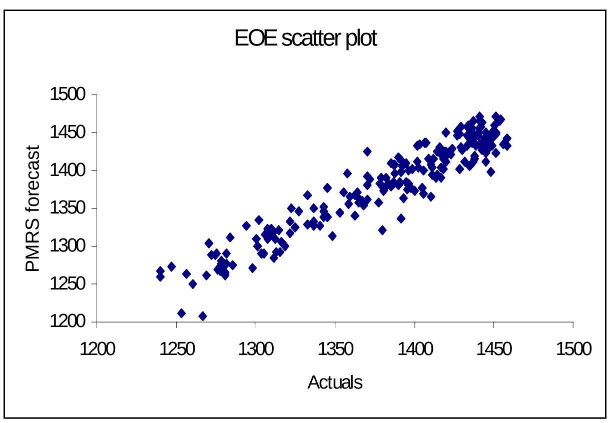
Appendix II. Scatter plots for PMRS forecasts

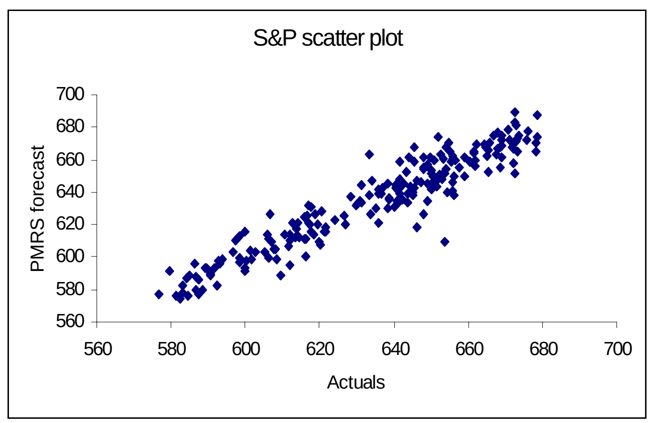












APPENDIX III. Error measures

m	is the forecasting method
rw	is the random walk method
s	is the series being forecast
$F_{\mathrm{m,s}}$	is the forecast from method m for series s
$A_{\mathrm{m,s}}$	is the actual value for series s

Root mean square error per forecast

$$RMSE_m = 1/N\left(\sum_{n=1}^{N}(F_{m,s} - A_s)^2\right)^{1/2}$$

Mean Absolute Percentage Error

$$MAPE_m = 100/N\left(\sum_{n=1}^{N}|(F_{m,s} - A_s)|\right)/A_s$$

Geometric Root Mean Square Error per forrecast

$$GRMSE_m = 1/N\left(\prod_{n=1}^{N}(F_{m,s} - A_s)^2\right)^{\frac{1}{2N}}$$

Relative Absolute Error

$$RAE_m = \frac{|F_{m,s} - A_s|}{|F_{rw,s} - A_s|}$$

Geometric Relative Absolute Error per forecast

$$GMRAE_m = 1/N\left(\prod_{n=1}^{N}(RAE_m)^{\frac{1}{N}}\right)$$