Back to basics: historical option pricing revisited

Jean-Philippe Bouchaud[1,2], Marc Potters[2]

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[1] Service de Physique de l'État Condensé,
Centre d'études de Saclay, Orme des Merisiers,
91191 Gif-sur-Yvette Cédex, FRANCE
E-mail: bouchau@amoco.saclay.cea.fr
[2] Science & Finance, 109-111 rue Victor Hugo,
92523 Levallois Cedex, FRANCE
E-mail: marc.potters@science-finance.fr

Abstract

We reconsider the problem of option pricing using historical probability distributions. We first discuss how the risk-minimisation scheme proposed recently is an adequate starting point under the realistic assumption that price increments are uncorrelated (but not necessarily independent) and of arbitrary probability density. We discuss in particular how, in the Gaussian limit, the Black-Scholes results are recovered, including the fact that the average return of the underlying stock disappears from the price (and the hedging strategy). We compare this theory to real option prices and find these reflect in a surprisingly accurate way the subtle statistical features of the underlying asset fluctuations.

1 Introduction

The famous Black and Scholes option pricing theory has two remarkable features: the hedging strategy eliminates risk entirely, and the option price does not depend at all on the average return of the underlying asset [1, 2, 3]. The second property means that the option price is not simply the actualized average of the future pay-off over the historical probability distribution, which obviously would depend on the average return. This is even more striking in the case of the Cox-Ross-Rubinstein binomial model [4, 2] where the pricing measure is completely unrelated to the actual distribution of returns. This has lead to a rather abstract and general framework for derivative pricing, where the absence of arbitrage opportunities leads, for models where risk can be eliminated completely, to the existence of a 'risk-neutral probability measure' (unrelated to the historical one) over which the relevant average should be taken to obtain the price of derivatives [5, 6]. It is thus a rather common belief that the knowledge of the 'true' probability distribution of returns is a useless information to price options. The credence is rather that the relevant 'implied' value of the parameters should be obtained from option market themselves, and used to price other instruments (for example exotic options) [2, 7].

However, in most models of stock fluctuations, except for very special cases (continuous time Brownian motion and binomial, both being very poor representation of the reality), risk in option trading cannot be eliminated, and strict arbitrage opportunities do not exist, whatever the price of the option. That risk cannot be eliminated is furthermore the fundamental reason for the very existence of option markets. It would thus be more satisfactory to have a theory of options where the true historical behaviour of the underlying asset was used to compute the option price, the hedging strategy, and *the residual risk*. The latter is clearly important to estimate, both for risk control purposes, but also because it is reasonable to think that this residual risk partly determines the bid-ask spread imposed by market-makers. The natural framework for this is the risk minimisation approach developed by several authors [8, 9, 10, 11, 12, 13, 14], where the optimal trading strategy is determined such that the chosen measure of risk (for example the variance of the wealth balance) is minimized. The 'theoretical' price is then obtained using a fair game argument. Note that in this approach, the option price is not unique since it depends on the definition of risk; furthermore, a risk-premium correction to the fair game price can be expected in general. From a theoretical point of view, this would generally be regarded as a lethal inconsistency. From a practical point of view, however, we see this as an advantage: since the price ambiguity is a constitutive property of option markets, it is interesting to understand the origin and size of this ambiguity. In this framework, the historical probability distribution determines the 'pricing kernel' to be used in the option price formula. We show in detail how, in the Black-Scholes limit, the average trend indeed completely disappears from the formula, and all the classical results are recovered. For more general models, however, the independence of the price on the average return is non trivial.

The outline of this paper is as follows. We first recall the basic steps leading to option prices and optimal hedges for a general process with uncorrelated (but not necessarily independent) increments, which we present in terms of a cumulant expansion to show how the Black- Scholes results are obtained in the corresponding Gaussian limit. The first cumulant correction provides a theory for the volatility smile in terms of the (maturity dependent) kurtosis of the terminal price distribution. We compare this theory to real option prices (on a liquid market) and find that these option prices reflect in a surprisingly accurate way the subtle statistical features of the underlying asset fluctuations [15, 16], in particular the persistent nature of the volatility fluctuations.

2 A risk minimisation theory of option pricing

2.1 The global wealth balance

Let us first write the wealth balance equation corresponding to the writing of a European call option. At time $t = 0$, the writer receives the price of the option $\mathcal{C}[x_0, x_s, T]$, on a certain asset which value is $x(t = 0) = x_0$. The strike price is x_s. Between $t = 0$ and $t = T$, the writer trades the underlying asset at discrete times $t = k\tau,\ k = 1, ..., N = T/\tau$; his strategy is to hold $\phi_k(x_k)$ assets if the price is $x(t) = x_k$ when the time is $t = k\tau$. It is easy to show that the change of wealth due to this trading is given by [11]:

$$\Delta W_{\text{trading}} = \sum_{k=0}^{N-1} \phi_k(x_k)[x_{k+1} - e^{r\tau} x_k] e^{r(T-t_k-\tau)} \tag{1}$$

where r is risk-free rate and $t_k = k\tau$. At time $T = N\tau$, the writer looses the difference $x_N - x_s$ if the option is exercized. Thus the complete wealth balance reads:

$$\Delta W = \mathcal{C}[x_0, x_s, T]e^{rT} - \max(x_N - x_s, 0) + \sum_{k=0}^{N-1} \phi_k(x_k)\delta x_k e^{r(T-t_{k+1})} \tag{2}$$

where we have introduced the notation: $\delta x_k \equiv [x_{k+1} - e^{r\tau} x_k]$. Note that δx_k is posterior to the instant k where ϕ_k is determined. Denoting as $\langle ... \rangle$ the average over the historical distribution, the average profit is given by:

$$\langle \Delta W \rangle = \mathcal{C}[x_0, x_s, T]e^{rT} - \langle \max(x_N - x_s, 0) \rangle + \sum_{k=0}^{N-1} \langle \phi_k(x_k) \rangle \langle \delta x_k \rangle e^{r(T-t_{k+1})} \tag{3}$$

The fair game requirement then fixes $\mathcal{C}[x_0, x_s, T]$ such that $\langle \Delta W \rangle = 0$.

Although other interesting definitions could be considered [11], we restrict here to the case where the risk is measured as:

$$R \equiv \langle \Delta W^2 \rangle - \langle \Delta W \rangle^2 = \langle \Delta W^2 \rangle \tag{4}$$

The risk R is always greater than or equal to zero and the minimum is obtained for a certain optimal strategy ϕ^*, determined by a functional derivation of (4) with respect to with respect to $\phi(x, t)$. This determines the option price through:

$$\mathcal{C}[x_0, x_s, T] = e^{-rT} \langle \max(x_N - x_s, 0) \rangle - \sum_{k=0}^{N-1} \langle \phi_k^*(x_k) \rangle \langle \delta x_k \rangle e^{-rt_{k+1}} \tag{5}$$

Note that since ϕ^* depends a priori on our choice of the variance as the relevant measure of risk, the price of the option is not unique, but reflects (among other things) the operator's perception of risk.

2.2 The case of zero excess average return

In this section we consider the case where the average return of the stock over the bond, $m = \langle \delta x_k \rangle$, is zero. This simplifying hypothesis is often justified in practice for small maturities, where average return effects are small compared to volatilites, and can be treated perturbatively, as shown in section 2.3.

Let $P(x, T|x_0, 0)dx$ be the probability that the asset value is x at time T, knowing that it was x_0 at time 0. When $m = 0$, Eq. (5) then yields an option price independent of the trading strategy:

$$\mathcal{C}[x_0, x_s, T; m = 0] = e^{-rT}\langle \max(x_N - x_s, 0)\rangle \equiv e^{-rT}\int_{x_s}^{\infty} dx(x - x_s)P(x, T|x_0, 0) \tag{6}$$

Note that our assumption that $m = 0$ means that the average of $P(x, T|x_0, 0)$ is not at x_0, but at the forward price $x_0 e^{rT}$.

In order to proceed with the risk-minimization, we shall assume that the price increments δx_k are uncorrelated random variables, such that $\langle \delta x_k \delta x_\ell \rangle = \sigma^2 \delta_{k,\ell}$, where $\delta_{k,\ell}$ is the Kronecker symbol. Assuming that σ does not depend on k is in general not justified, since it amounts to assuming that share price follows an additive random process of constant volatility (but with an arbitrary distribution for the increments). Actually, real data is often closer (for short maturities) to being an additive random process rather than a multiplicative one [11], an assumption which does introduce an spurious positive skew in the price distribution. In reality, however, σ depends on k, which reflects ARCH-like effects (or time persistent volatility [15]). Taking this effect into account would lead to more involved calculations, which can however still be completed analytically [11]. With these approximations in mind, the relevant formula for risk is rather simple:

$$\begin{aligned}
\langle \Delta W^2 \rangle &= \langle \Delta W^2 \rangle_0 + \sigma^2 \sum_{k=0}^{N-1} \int_0^\infty dx P(x, t_k|x_0, 0)\phi_k^2(x) e^{2r(T - t_{k+1})} \\
&- 2e^{r(T - t_{k+1})} \sum_{k=0}^{N-1} \int_0^\infty dx P(x, t_k|x_0, 0)\phi_k(x) \\
&\quad \int_{x_s}^\infty dx'(x' - x_s)P(x', T|x, t_k)\langle \delta x_k \rangle_{x, t_k \to x', T}
\end{aligned} \tag{7}$$

where $\langle \Delta W^2 \rangle_0$ is the unhedged ($\phi_k \equiv 0$) risk associated to the option, and $\langle \delta x_k \rangle_{x, t_k \to x', T}$ is the conditional average of δx_k, on the trajectories starting at x at time t_k and ending at point x' at time T.

The optimal trading strategy is obtained by setting [10, 11, 12]:

$$\frac{\partial \langle \Delta W^2 \rangle}{\partial \phi_k(x)} = 0 \tag{8}$$

for all k and x. This leads to the following explicit result for the optimal hedging strategy:

$$\phi_k^*(x) = \frac{e^{-r(T-t_{k+1})}}{\sigma^2} \int_{x_s}^{\infty} dx'(x'-x_s) \langle \delta x_k \rangle_{x,t_k \to x',T} P(x',T|x,t_k) \tag{9}$$

This formula simplifies somewhat when the increments are Gaussian, and one finally finds the famous Black-Scholes '$\Delta-$hedge': $\phi_k^*(x) = \partial \mathcal{C}[x, x_s, T - t_k]/\partial x$. In the non-Gaussian case, however, this simple relation between the derivative of the option price and the trading strategy no longer holds (see Eq. (13) below).

Inserting (9) into (7) leads to the following formula for the residual risk:

$$R^* = \langle \Delta W^2 \rangle_0 - D\tau \sum_{k=0}^{N-1} \int_0^{\infty} dx P(x,t_k|x_0,0) \phi_k^{*2}(x) e^{-r(T-t_{k+1})} \tag{10}$$

In general, the left-hand side of (10) is non-zero; in practice it is even quite high – for example, for typical one-month options on liquid markets, $\sqrt{R*}$ represents as much as 25% of the option price itself [11]. However, in the special case where $P(x,t|x_0,0)$ is normal (or log-normal), and in the limit of continuous trading, that is, when $\tau \to 0$, one can show that the residual risk R^* actually vanishes, thanks to a somewhat miraculous identity for Gaussian integrals [10]. Hence, the above formalism matches smoothly with all the Black-Scholes results in the limit of a continuous time Brownian (or log-Brownian) process, at least when the excess average return of the asset is zero. Let us now discuss how these results are changed if the average return $m \equiv \langle \delta x_k \rangle$ is non zero (but small).

2.3 Small non-zero average return

More precisely, we shall consider the case where $mN \ll \sigma\sqrt{N}$ ($N = T/\tau$), or, more intuitively, that the average return on the time scale of the option is small compared to the typical variations, which is certainly the case for options up to a few months[1]. The global wealth balance then includes the term related to the trading strategy, which reads[2]:

$$\langle \Delta W_{\text{trading}} \rangle = m \sum_{k=0}^{N-1} \int_0^{\infty} dx P(x, t = k\tau | x_0, 0) \phi_k^*(x) \tag{11}$$

The advantage of considering a small average return is that one can do a perturbation around the zero average return case, and still use the explicit optimal strategy of (9) to lowest order in m.

Compared to the case $m = 0$, the option price is changed both because $P(x, k|x_0, 0)$ is *biased*, and because $\langle \Delta W_{\text{trading}} \rangle$ must be substracted off from Eq. (5). It is convenient to use the Fourier transform of the probability distribution $\tilde{P}(z) = \int_{-\infty}^{\infty} dx P(x, N|x_0, 0) \exp(ixz)$ and to expand it in a series introducing the *cumulants* c_n. They are defined by

$$\tilde{P}(z) = \exp\Big[\sum_{n=1}^{\infty} \frac{c_n (iz)^n}{n!}\Big], \tag{12}$$

where $c_1 = mN$, $c_2 = N\sigma^2$, $\kappa = c_4/c_2^2$ is the kurtosis, etc... Applying the cumulant expansion to the probability distribution in Eq.(9), we obtain the following optimal strategy for $m = 0$ [11]:

$$\phi_k^*(x) = \frac{1}{c_2} \sum_{n=2}^{\infty} \frac{(-1)^n c_n}{(n-1)!} \frac{\partial^{n-1}}{\partial x^{n-1}} \mathcal{C}[x, x_s, T - t_k]. \tag{13}$$

Note again that in the Gaussian case ($c_n = 0$ for $n > 2$), one recovers the standard Black and Scholes '$\Delta -$hedge'.

Inserting the optimal strategy (13) into Eq.(11) and integrating by parts one gets an expansion of the trading term $\langle \Delta W_{\text{trading}} \rangle$. Inserted into Eq.(5), this gives the following correction to the option price [11]:

$$\begin{aligned} \mathcal{C}[x_0, x_s, T; m] &= \mathcal{C}[x_0, x_s, T; m = 0] \\ &\quad - \frac{m}{c_2} \sum_{n=3}^{\infty} \frac{c_n}{(n-1)!} \frac{\partial^{n-3}}{\partial x'^{n-3}} P_0(x', N|x_0, 0)|_{x'=x_s} + \mathcal{O}(m^2) \end{aligned} \tag{14}$$

[1]Typically, $m = 5\%$ annual and $\sigma = 15\%$ annual. The order of magnitude of the error made in neglecting the second order term in m is $m^2 N/\sigma^2 \simeq 0.1$ even for $N\tau = 1$ year.

[2]For the sake of simplicity, we shall set the interest r to zero in the following. See [12] for a more complete discussion.

where the shorthand P_0 stands for the probability distribution where the first cumulant m has been set to zero.

In the Gaussian case, $c_n = 0$ for all $n \geq 3$, and one thus sees explicitly that $\mathcal{C}_m = \mathcal{C}_0$, at least to first order in m. Actually, one can show that this is true to all orders in m in the Gaussian case, which is an alternative way to derive the result of Black and Scholes in a Gaussian context [11] [3].

However, for even distributions with fat tails ($c_3 = 0$ and $c_4 > 0$), it is easy to see from the above formula that a positive average return $m > 0$ increases the price of out-of-the-money options ($x_s > x_0$), and decreases the price of in-the-money options ($x_s < x_0$). Hence, we see again explicitly that the independence of the option price on the average return m, which is one of the most important result of Black and Scholes, does not survive for more general models of stock fluctuations.

Note finally that Eq. (15) can also be written as:

$$\mathcal{C}[x_0, x_s, T; m] = \int_{x_s}^{\infty} dx(x - x_s)Q(x, T|x_0, 0) \tag{15}$$

with an effective distribution Q defined as:

$$Q(x, T|x_0, 0) = P_0(x, T|x_0, 0) - \frac{m}{c_2} \sum_{n=3}^{\infty} \frac{c_n}{(n-1)!} \frac{\partial^{n-1}}{\partial x^{n-1}} P_0(x, N|x_0, 0) \tag{16}$$

The integral over x of Q is one, but Q is not a priori positive everywhere. This means that for a certain family of pay- offs, the fair price of the option (15) may be negative in the absence of risk-premium. From a practical point of view, however, this requires rather absurd values for the average return and for the strike price, which in turn would lead to a large residual risk.

The pseudo-distribution Q generalizes the 'risk neutral probability' usually discussed in the context of the Black-Scholes theory, and also has the property that the excess average return (the integral of $(x - x_0)Q$ over x) is zero, as can easily be seen by inspection from (16). In fact, one can derive a general formula for Q without any restriction on m of r [12, 13], and the effective distribution still has the properties that it is normalized to one and has zero average excess return.

[3] This result is obvious on the framework of Ito's calculus, which is only valid for all Gaussian processes (including the log-normal) in the continuum time limit.

3 Volatility smile and implied kurtosis

In the case where the market fluctuations are moderately non-Gaussian, as is the case for liquid markets, one might expect that the first terms in the cumulant expansion around the Black-Scholes formula are sufficient to account for real option prices. If one only retains the leading order correction which is (for symmetric fluctuations) proportional to the kurtosis κ, one finds that the price of options $\mathcal{C}(x_0, x_s, T)$ can be written as a Gaussian Black-Scholes formula[4], but with a modified value of the volatility σ, which becomes price and maturity dependent [17]:

$$\sigma_{\text{imp}}(x_s, T) = \sigma \left[1 + \frac{\kappa_T}{24} \left(\frac{(x_s - x_0)^2}{\sigma^2 T} - 1 \right) \right] \tag{17}$$

The volatility σ_{imp} is called the implied volatility by the market operators, who use the standard Black-Scholes formula to price options, but with a value of the volatility which they estimate intuitively, and which turns out to depend on the exercice price in a roughly parabolic manner, as indeed suggested by Eq. (17). This is the famous 'volatility smile'. Eq. (17) furthermore shows that the curvature of the smile is directly related to the kurtosis κ_T of the underlying statistical process on the scale of the maturity $T = N\tau$. We have tested this prediction by directly comparing the 'implied kurtosis' [18], obtained by extracting from real option prices the volatility σ and the curvature of the implied volatility smile, to the historical value of the volatility and of the kurtosis κ_N. We have mostly studied short maturities (up to two months) options on futures, for which the interest rate can be set to zero. We also restrict to liquid markets (such as the BUND option market) where (i) non Gaussian effects are not too strong, and (ii) risk-premiums are expected to be small, and thus where a comparison with the fair price is meaningful.

We have found the following results. The implied volatility turns out to be highly correlated with a short time filter of the historical volatility: see Fig. 1. Fig. 2 shows the comparison between the implied and historical kurtosis, *with*

[4] Note that the operators rather use the more standard log-normal Black-Scholes formula, which, as noted above, induces a spurious positive skew not present in real data (at least for short maturities). In order to correct for this skew, the log-normal volatility smile is then negatively skewed. A more symmetric smile is observed if one talks in terms of a Gaussian volatility, which is what we adopt in the following.

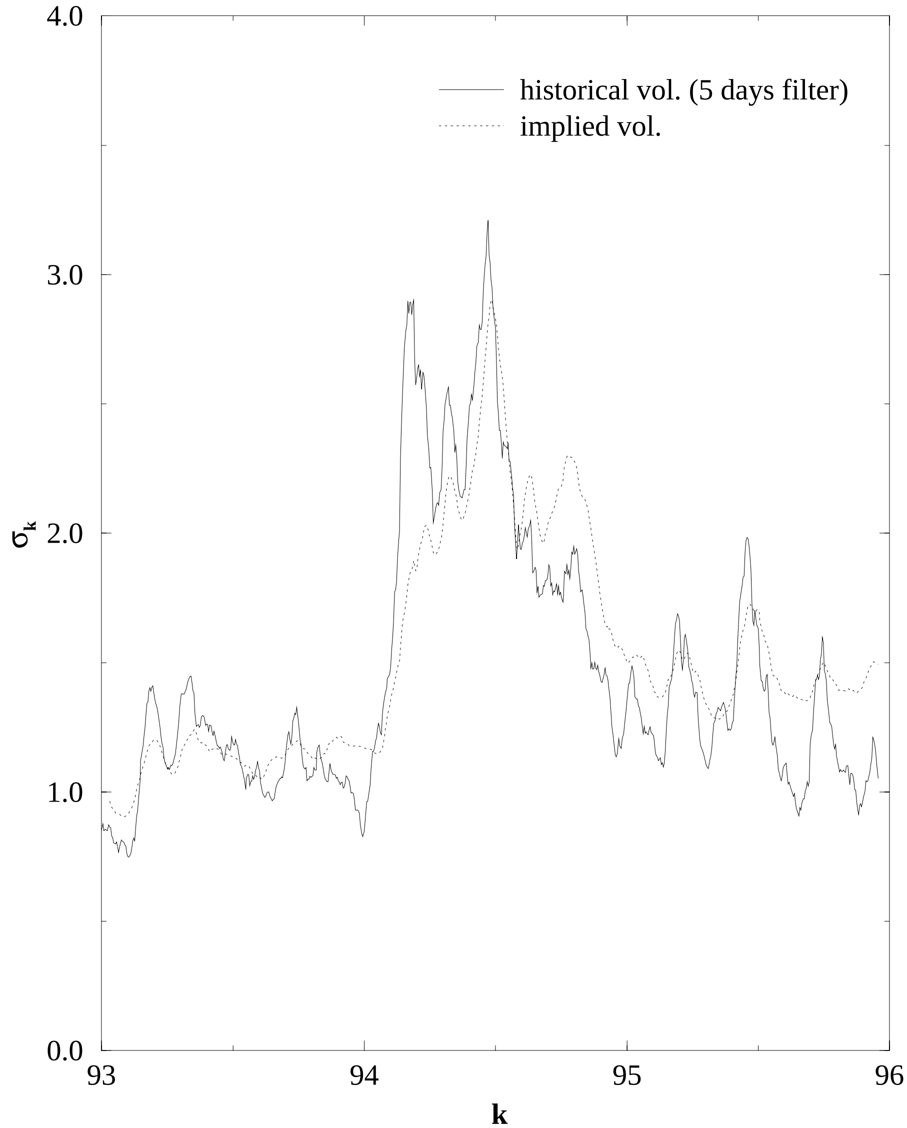

Figure 1: Comparison between the historical volatility of the BUND (measured from high frequency data and filtered over the past five days), and the implied volatility, extracted from the option prices through formula (17).

no further ajustable parameters. Note that the historical kurtosis decays more slowly than N^{-1}, which would be expected for a process with independent, identically distributed increments. This anomalously slow decay is directly related to volatility persistence effects [17, 11].

It is interesting to note that the kurtosis correction to the optimal strategy does not coincide with the market practice of using the implied volatility in the Black-Scholes $\Delta-$hedge. However, since the risk is minimum for $\phi = \phi^*$, this means that the increase of risk due to a small error $\delta\phi$ in the strategy is only of order $\delta\phi^2$, and thus often quite small in practical applications.

The remarkable agreement between the implied and historical value of the parameters (which we have also found on a variety of other assets), and the fact that they evolve similarly with maturity, shows that the market as a whole is able to correct (by trial and errors) the inadequacies of the Black-Scholes formula, and to encode in a satisfactory way both the fact that the distribution has a positive kurtosis, and that this kurtosis decays with maturity in an anomalous fashion due to volatility persistence effects.

4 Conclusion

In our opinion, mathematical finance in the past decades has overfocused on the concept of arbitrage free pricing, which relies on very specific models (or instruments) where risk can be eliminated completely. This leads to a remarkably elegant and consistent formalism, where derivative pricing amounts to determining the risk-neutral probability measure, which in general does not coincide with the historical measure. In doing so, however, many important and subtle features are swept under the rug, in particular the amplitude of the residual risk. Furthermore, the fact that the risk-neutral and historical probabilities need not be the same is often an excuse for not worrying when the parameters of a specific model deduced from derivative markets are very different from historical ones. This is particularly obvious in the case of interest rates [19]. In our mind, this rather reflects that an important effect has been left out of the models, which in the case of interest rates is a risk premium effect [19]. We believe that a more versatile (although less elegant from a mathematical point of view) theory of derivative pricing, such as the one discussed above, allows one to use in a consistent and fruitful way the empirical data on the underlying asset to price, hedge, and control the risk

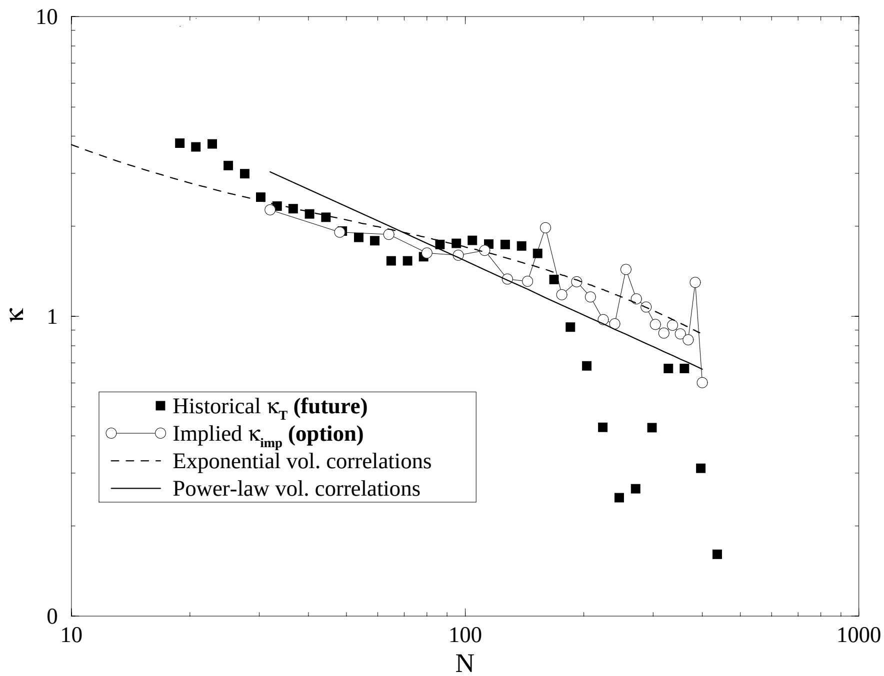

Figure 2: Plot (in log-log coordinates) of the average implied kurtosis κ_{imp} (determined by fitting the implied volatility for a fixed maturity by a parabola) and of the empirical kurtosis κ_N (determined directly from the historical movements of the BUND contract), as a function of the reduced time scale $N = T/\tau$, $\tau = 30$ minutes. All transactions of options on the BUND future from 1993 to 1995 were analyzed along with 5 minute tick data of the BUND future for the same period. We show for comparison a fit with $\kappa_N \simeq N^{-0.6}$ (dark line). A fit with an exponentially decaying volatility correlation function is however also acceptable (dotted line).

the corresponding derivative security. Extension of these ideas to interest rate derivatives is underway.

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