

**International CAPM with Regime Switching  
GARCH Parameters**

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# International CAPM with Regime Switching Parameters: Executive Summary

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When investing in a risky asset, the investor is uncertain about its return, and will, if risk averse, require a market risk premium. Moreover, if the asset is denominated in a foreign currency, the investor must also bear the exchange rate risk, and should demand an additional exchange rate premium. This paper seeks to establish better estimates of the time-varying market and currency risk premia in the major international stock and currency markets (USA, Europe and Japan). Risk premia can be decomposed into a market (currency) price of risk multiplied by the corresponding non-diversifiable market (currency) risk exposure, as measured by the covariance between excess asset and market (currency) returns. The prices of risk provide a measure of the coefficients of risk aversion.

The model we use to measure risk premia in a financially integrated world is the International Capital Asset Pricing Model (CAPM) of Adler and Dumas (1983). The estimation employs a multivariate GARCH(1,1)-in-Mean specification of the model.

Other authors, too, have recently estimated the International CAPM for selected large-capitalization countries. Our work differs from these earlier analyses in both data selection and methodology. First, we use weekly rather than monthly data, which allows us to focus on a recent period, 1986 to 1998, which is characterized by growing financial integration. Secondly, we employ an aggregate stock market index for Europe, thereby obtaining a parsimonious model specification which encompasses fully 95% of the world market capitalization. Finally, and more importantly, we allow the GARCH parameters to change value over time according to a Markovian regime-switching process. Previous research has documented that structural changes in the GARCH parameters, if not modelled, give rise to spuriously high persistence of conditional volatility, as well as poor forecasting performance. In other words, traditional non-switching GARCH models tend to underestimate the conditional volatility at the time when it goes from a low to high state, and, similarly, overestimate it when volatility changes from a high back to a low state. By letting the GARCH parameters switch, we thus expect to obtain improved estimates of the conditional volatilities (and covariances). A switching GARCH also has an appealing economic justification, as it directly addresses Lucas' critique of structural models with constant parameters.

Our estimation yields both *time* and *state*-varying market and currency risk exposures, as well as their associated prices of risk. The latter are assumed to be constant to make estimation tractable. As benchmark, the International CAPM is estimated with a traditional, non-switching multivariate GARCH(1,1)-in-Mean methodology. For this benchmark model, we find that the price of market risk is significant, in contrast to earlier results obtained with International CAPM incorporating currency premia and

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constant prices of risk. Currency risks, however, are not significantly priced. All major market turmoils are captured, as reflected by increases of risk premia. For the switching-GARCH model, on the other hand, the price of market risk is no longer found to be significant. However, and more importantly, stock market risk exposures are seen to react faster to shocks in returns. This may suggest that the non-switching model indeed suffers from spuriously high persistence, with consequently sluggish reaction to shocks. In particular, when a financial crisis occurs, the conditional risk exposures seem to be underestimated, while in the aftermath they appear to be overestimated. The inclusion of a regime switching GARCH should hence improve forecasting performance, a result that is in line with previous research on switching (G)ARCH processes.

We also find that in periods of financial turmoil, weight is shifting away from the GARCH towards the ARCH terms of the conditional (co)variance generating process. This result leads to an appealing and intuitive interpretation of investors' "crisis mentality": When forming their volatility (and covariance) expectations in periods of crises, investors are less inclined to extrapolate past expectations into the future, while more inclined to take into account current shocks when updating their expectations. In other words, their memory of the past fades, while concern about the present rises.

The methodology presented in this paper has practical applications in global asset management. Since we estimate the conditional covariance matrix and expected returns, as well as the (constant) prices of risk, time varying optimal portfolio weights can be calculated. Consequently, a global portfolio can be re-optimized each period, allowing for dynamic portfolio management. The novelty we introduce, however, lies in the use of switching GARCH parameters. Hence, different regimes will produce different sets of optimal portfolio weights, according to the state of the conditional covariance matrix. Such state-dependent optimal portfolio weights should adjust faster to shocks in financial markets than those calculated with a traditional non-switching model. Moreover, our methodology can also provide state dependent, and thus better, measures of expected gains from cross-border diversification, optimal hedge ratios, and conditional beta and correlation coefficients.

# International CAPM with Regime Switching GARCH Parameters\*

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## Abstract

This paper tests a conditional version of Adler and Dumas' (1983) International CAPM with regime switching GARCH parameters. As benchmark the same model is estimated without state dependent parameters. The switching representation is found to react faster than the benchmark to shocks in stock market returns. This suggests that the non-switching model suffers from spuriously high persistence. In particular, when a financial crisis occurs, the conditional risk exposures appear to be underestimated, while overestimated in the aftermath. The introduction of a regime switching model should hence improve forecasting power. We also find that in periods of financial turmoil, weight is shifting from the GARCH towards the ARCH terms of the conditional covariance generating process. During such events investors, when forming their (co)variance expectations, seem to put more emphasis on current shocks, at the expense of the current second moments.

**Keywords:** International CAPM; Multivariate GARCH-in-Mean; Regime Switching.

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# 1 Introduction

When agents invest in a risky asset, they are uncertain about its return, hence the need for a risk premium. According to the Capital Asset Pricing Model (CAPM) the expected risk premium is proportional to the undiversifiable risk exposure, which is the covariance between the asset return and the return of the market portfolio. The proportionality factor is the price of risk, which equals the risk aversion. If, on the other hand, the asset is denominated in a foreign currency, the investor must also bear the foreign currency risk. Again, if the investor is risk averse, he should require both a market and an exchange rate risk premium. However, it has been difficult to measure the magnitude of exchange rate risk premia, and this is indeed one of the persistent questions in empirical international finance and macroeconomics (see, for instance, Hodrick 1987; Lewis 1995; and Engle 1995).

The goal of this paper is to estimate, in a *stochastic* fashion, both market and currency risk premia for a selected group of developed capital markets. We adopt a methodology which should capture the observed volatility clustering in asset returns, as well as any sudden shifts in the data generating process of the conditional (co)variances.

## 1.1 Literature Review and Research Goals

The last decade of research in international empirical finance has shown that expected returns on, say, national equity indices cannot be solely explained by their covariation with the world market returns. The extension of the domestic static CAPM to an international environment does not validate the model, possibly indicating the need for more than one source of risk. Several examples can be quoted: Giovannini and Jorion (1989), applying the traditional CAPM to international markets, find the price of world market risk not to be significantly different from zero; Korajczyk and Viallet (1990) provide evidence that multifactor models tend to outperform single-factor CAPM-type models; Harvey (1991) and Ferson and Harvey (1993) show that time-varying covariances between excess returns on national indices and the world market are unable to explain all the dynamic behavior of the country returns; Chan, Karolyi, and Stulz (1992) find that an enriched CAPM is not rejected, which, however, is probably due to the inclusion of extra priced factors that take into account the lack of synchronism in trading hours; Bekaert and Harvey (1995), testing the degree of market integration, show that the CAPM is rejected for most of the considered countries; Bekaert and Harvey (1997) conclude that equity volatility of some emerging markets is affected by world and local factors; De Santis and Gerard (1997), when

restricting the price of market risk to be positive, find that a one-factor model is unable to fully explain the dynamics of international expected returns; and Oertmann (1997), estimating a conditional multi-beta asset pricing model, shows that the performance of a global portfolio with stocks and bonds is exposed to three sources of global risk: the return on the world stock market, changes in the level of global interest rates, and variations in exchange rates.

The first issue we address, therefore, is the need for extra priced risk factors in CAPM. We include exchange rate premia, and thereby focus on one possible set of factors which emerge in an international environment. Our theoretical model is the International CAPM of Adler and Dumas (1983). Thus, the research fits into the recent strand of literature mentioned above. Of course, the exchange rate risk is only one of many possible additional priced risk factors, but we do not consider other candidates. Although other researchers (Dumas and Solnik 1995; De Santis and Gerard 1998; De Santis, Gerard and Hillion 1998; and Cappiello 1998) have already estimated the above mentioned International CAPM, our study differs in two important ways: First, in the choice of data set and observation frequency (we analyze USA, Europe *as a whole*, and Japan at a *weekly* frequency, whereas the other studies look at *individual* countries at a *monthly* frequency). Second, and more importantly, we propose a novel estimation method which allows for stochastic shifts in the model parameters, which we now turn to.

A common way to model conditional second moments implied by CAPM is through a (Generalized) Autoregressive Conditional Heteroskedastic -in-Mean ((G)ARCH-M) specification. However, it has been pointed out that a GARCH parametrization suffers from a number of related drawbacks. Estimated GARCH models typically show a very high persistence parameter (close to one), which led Engle and Bollerslev (1986) to introduce the integrated-GARCH (I-GARCH), in which shocks to variance do not decay with time<sup>1</sup>. Diebold (1986) was probably the first to suggest that such high persistence might be due to shifts in the constant term of the GARCH process, not accounted for by the econometrician. Such shifts could be caused by, for instance, low-probability events like the 1987 stock market crash, and would manifest themselves as jumps in the dependent variable of the mean equation. Following up on this suggestion, Lamoureux and Lastrapes (1990) explicitly introduced deterministic shifts in the constant term through dummy variables. They discovered that, for long time series, the

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<sup>1</sup>In a univariate GARCH(1,1) the conditional variance is:  $h_{t+1} = c + a\varepsilon_t^2 + bh_t$ , where  $\varepsilon_t$  is the error term in a mean equation, and is defined as  $\varepsilon_t = \sqrt{h_t}\nu_t$ , with  $\nu_t$  being i.i.d. with zero mean and unit variance. The persistence parameter is then  $\lambda = a + b$ . To see this, define the conditional variance innovation as  $u_t = \varepsilon_t^2 - h_t$ . One then obtains  $h_{t+1} = c + \lambda h_t + au_t$ , which shows that  $\lambda$  measures the persistence of the process.

measured persistence dropped significantly. Engle and Mustafa (1992) indeed showed that the implied volatility fell much faster after the 1987 stock market crash than a GARCH model would suggest.

Hamilton and Susmel (1994) and Cai (1994) addressed this issue of (G)ARCH parameter instability by modelling the shifts of a univariate ARCH process in a *stochastic* fashion, using the methodology of Hamilton's (1988, 1989, 1990, 1994) regime switching model. The advantage of this procedure, relative to the above mentioned deterministic approach, is that it "endogenizes the parameter shifts and lets the data decide at what time a sudden dramatic event has occurred" (Cai 1994). Hamilton and Susmel let a scaling factor, which multiplies the conditional variance of ARCH-modelled stock prices, account for the switching. They find that the persistence of second moments decreases, and that both the statistical fit to the data and the forecastability in- and out-of-sample improve. Thus, structural changes in the (G)ARCH parameters, if not modelled, cause not only spuriously high persistence, but also poor forecasting performance. If the high persistence of traditional GARCH models is in fact spurious, it means that a GARCH will underestimate the conditional volatility at the time when it goes from a low (or normal) to a high state, and, similarly, overestimate it when volatility changes from a high back to a low (or normal) state. In other words, spuriousness breaks the link, which may exist when the mean equation is adequately specified, between high persistence and good forecasting capability. By combining a (G)ARCH with a regime switching model the spurious persistence should disappear, and the forecasting performance should improve. Cai (1994) introduces switching in the constant term of an ARCH specification, when estimating excess returns of 3-month T-bills. Again, the persistence of the ARCH process is reduced. Gray (1996) adopts a regime switching univariate GARCH, and finds again an improvement in forecasting performance. Dueker (1997) studies various univariate switching GARCH specifications, and identifies certain regime switching models with superior forecasting capability compared with that of a conventional GARCH. Ramchand and Susmel (1998a) build up an AR(1) switching ARCH model for weekly stock returns, in the spirit of Hamilton and Susmel. Once more, the conditional variance turns out to be both time and state-varying, and in high-volatility states conditional correlations between international stock markets tend to increase.

Letting GARCH parameters switch also has an appealing economic justification, as it directly addresses Lucas' critique of forcing the parameters of structural models to be constant. It is not plausible that the parameters of the conditional covariance matrix stay constant over the last fifteen years. Just consider the fact that during this period financial markets have



been increasingly liberalized, derivatives for hedging and speculation have become commonplace, mutual funds and pension funds have mushroomed, ultrafast trading techniques have been introduced, and governments and central banks have changed monetary and exchange rate policies. It is natural to assume that such ongoing structural changes affect the way investors perceive risk and form their expectations. If ever Lucas' critique applies, it must be in the rapidly changing financial markets.

The second and main issue we focus on, therefore, is the estimation of the International CAPM with a *stochastic* multivariate GARCH-M specification. More precisely, we let the GARCH parameters switch according to Hamilton's (1988, 1989, 1990, 1994) regime switching model, where changes in regime are driven by a Markov latent variable. Using a switching GARCH, we capture not only the volatility clustering characteristic of financial time series, *but also* any discrete shifts of the underlying GARCH parameters. Such estimation is of direct interest to the current econometric debate about the persistence in GARCH processes and their forecasting power. Concretely, it will yield time and state-varying market *and* currency risk exposures, as well as their associated (constant) prices of risk, for a limited number of high-market-cap regions: the USA, Europe (represented by a basket stock market index), and Japan. Such a grouping encompasses around 95% of the world market capitalization at the end of 1998, and is, at the same time, highly parsimonious in terms of number of indices.

Significant progress has recently been made in the conditional estimation of the International CAPM. The research can be grouped into two strands: one approach has retained the currency risk premia, but modelled the conditional second moments with non-switching parameters. The other approach has not considered the currency risk premia, but has instead introduced regime switching parameters. The first strand is represented by, among others, Dumas and Solnik (1995), De Santis and Gerard (1998), De Santis, Gerard and Hillion (1998), and Capiello (1998). The empirical investigations of De Santis and Gerard, De Santis, Gerard and Hillion, and Capiello rely on a *traditional* multivariate GARCH-M estimation. They provide support for the International CAPM of Adler and Dumas (1983). Their results corroborate earlier evidence on the price of market and currency risk by Dumas and Solnik, whose estimation of International CAPM, based on Generalized Methods of Moments (GMM) rather than GARCH, shows that currency risk is priced and statistically significant. Thanks to their GARCH-M specification, De Santis and Gerard, De Santis, Gerard and Hillion, and Capiello are able to go further than that, as they manage to disentangle the market and exchange rate premia, and quantify their associated time-varying prices of risk and risk exposures. However, the analyses of

De Santis and Gerard, and De Santis, Gerard and Hillion are, intriguingly, not sensitive to the major events which have hit financial markets during the last two decades, such as the stock market crash of 1987 and the European exchange rate turmoil of 1992. Indeed, there is no clear sign of increased risk premia during these events, contrary to what one would expect. Cappiello estimates the International CAPM, also using a multivariate GARCH-M, for two countries (USA and Italy, plus, of course, the world equity portfolio). Interestingly, market and currency risk premia (as well as the associated prices) are here found to rise during periods of financial turmoil, notably during the 1987 and 1989 stock market crashes and the EMS crisis in Europe in 1992. The analysis differs from that of De Santis and Gerard, and De Santis, Gerard and Hillion in the specification of the GARCH process (less parsimonious, but more general) and in the number of countries (fewer countries).

The second strand of literature includes, for example, Bekaert and Harvey (1995) and Ramchand and Susmel (1998b). Bekaert and Harvey estimate an International CAPM to investigate the degree of market integration of selected emerging economies. They model second moments as a switching ARCH, which enter as regressors in the mean equation, so that the latter switches according to a latent variable. In Ramchand and Susmel it is the beta of the CAPM which is state dependent, while the residuals of the mean equation are modelled as an *ARCH* with a switching constant term. They show that beta is significantly different across low and high-variance states, with obvious implications for expected excess returns.

A regime switching bivariate CAPM has also been estimated for a *domestic* market by Santos (1999), where the GARCH-modelled covariance matrix is scaled by a regime-switching coefficient. He identifies three distinct volatility states in US securities returns.

Finally, there are other state dependent international asset pricing models, which, however, are not rooted in CAPM. Examples are Das and Uppal (1998) and Ang and Bekaert (1999). Das and Uppal model security returns as a jump-diffusion process, thereby capturing the fat tails and skewness caused by jumps in asset returns. Their estimation is used to calculate state dependent optimal portfolio weights. Ang and Bekaert have developed an international asset pricing model, assuming Constant Relative Risk Aversion (CRRA) utility function. They let stock returns be state dependent both in the mean equation and in the conditional variance, and find evidence for a high volatility-high correlation regime in international stock return, which tends to coincide with a bear market. State dependent optimal portfolio weights are also obtained.

## 2 Methodology

In the following three subsections we discuss the main results of Adler and Dumas' (1983) International CAPM, the multivariate GARCH specification used to model the second moments, and the method to combine the GARCH with Hamilton's (1988, 1989, 1990, 1994) regime switching model.

### 2.1 International CAPM

The theoretical model used is the conditional version of the International CAPM first developed by Adler and Dumas (1983). When agents invest in a foreign asset they have to bear a risk that comes from two sources: on one hand they are uncertain about the performance of the foreign asset; on the other hand, they don't know if the domestic currency is going to appreciate or depreciate against the foreign currency. International CAPM therefore contains additional terms to reward exchange rate risk, and differs from the classic CAPM inasmuch as the latter does not include such terms.

The only way to use the domestic CAPM in an international context is by making two unreasonable assumptions (Solnik 1997): i) Purchasing Power Parity (PPP) holds at any point in time, and, therefore, real prices of consumption goods are identical throughout the world. ii) Consumption baskets are the same for all investors in the world. If PPP were to hold, exchange rates would simply be determined by inflation differentials between two countries. In this case, no real exchange risk would exist and the exchange rate system would be a pure translation-accounting device. When, instead, investors of different countries have access to goods at different prices, one has the kind of heterogeneity indispensable to realistically represent an international world in which real returns from the same assets are seen differently. Since both deviations from PPP and differences in consumption preferences among countries are widely recognized, it's not possible to apply a domestic CAPM to integrated financial markets.

What is true for the domestic CAPM turns out also to be true for the International CAPM. A normative and a descriptive outcome therefore follows:

1. *Separation theorem.* A representative agent should hold a combination of two portfolios: A risky portfolio common to all investors, irrespective of their nationality, and a personalized portfolio to hedge purchasing power risk. The risky portfolio is simply defined as a share of the world market portfolio. Under the assumption of non-stochastic inflation (the so-called "Solnik's special case") the hedge portfolio reduces to the national risk-free asset. The intuition is straightforward. In the "Solnik's special case" what

is random is the local inflation rate measured in the reference currency (say US dollars). For example, the German inflation rate expressed in the domestic currency is not random *per se*, but once translated into dollars the randomness in the exchange rate becomes a factor that must be taken into account. As a consequence, the home currency bank deposits or Eurocurrency deposits are considered riskless in real terms by national investors and, as such, they will constitute the hedge portfolio. The small variability of inflation rates compared to that of exchange rates provides a justification for the “Solnik’s special case”.

2. *Risk pricing relation.* Unlike the domestic CAPM, expected returns depend not only on the market risk premium, but also on additional premia to hedge exchange rate risks.

Being more formal, let us assume there are  $L + 1$  fully integrated countries in the world, which amounts to  $L$  exchange rates against a reference currency. Let there be one risky asset (e.g. a stock market index) and one riskless short-term deposit (Eurocurrency deposit) per country. The latter, as already noted, is riskless only to the local investor. Take the view of a US investor, i.e. let the US dollar be the reference currency. The integrated world then contains  $L + 1$  risky stock market indices,  $L$  risky short-term currency deposits, and one risky world stock market portfolio. This gives  $2(L + 1)$  risky assets in total, whose expected excess return must be determined by an asset pricing model.

In International CAPM investors maximize lifetime utility of consumption<sup>2</sup>:

$$\max_{C, \omega} E \int_t^T V(C, P, \tau) d\tau, \quad (1)$$

subject to the budget constraint

$$dW = \left[ \sum_{i=1}^{2(L+1)} \omega_i (E(r_i) - r_f) + r_f \right] W dt - C dt + W \sum_{i=1}^{2(L+1)} \omega_i \sigma_i dz_i. \quad (2)$$

$E$  is the unconditional expectation operator,  $V(\cdot)$  is the instantaneous rate of indirect utility,  $C$  is the nominal rate of consumption expenditure,  $P$  is the price level index, which is included in the utility function in order to rule out money illusion (investors care about real wealth),  $\omega$  is the portfolio weights of the risky and riskless assets,  $W$  is nominal wealth of the investor,  $r_i$  is the return on risky asset  $i$ ,  $r_f$  is the return on the risk free

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<sup>2</sup>Time subscripts have been dropped to simplify the notation.

asset,  $z_i$  is a Wiener process which governs the stochastic component of the return on the risky asset  $i$ , and  $\sigma_i$  is its volatility. Hence investors choose optimal consumption and portfolio paths, subject to the budget constraint. Adler and Dumas assume a constant investment opportunity set, in the sense that  $E(r_i)$ ,  $r_f$  and  $\sigma_i$  are constant. In other words, the model was defined unconditionally, and has only recently been estimated conditionally. The currency risk premia here do not result from state variables, as in Merton's (1973) intertemporal CAPM, but rather they appear because international investors are concerned about *real* returns. A *conditional* International CAPM should, strictly speaking, be intertemporal, but this is beyond the scope of our research. Hence one might say that a conditional estimation of Adler and Dumas' International asset pricing model omits the underlying state variables.

The maximization yields an optimal portfolio path for each investor (the investment universe comprises all assets in the world). The total world demand for assets in each time period can then be found by aggregating over all investors in the world. The model then assumes that the supply of assets is exogenously given. Equating aggregate supply and demand, the model derives the expected excess return measured in reference currency  $c$ ,  $E(R_i^c)$ , which investors require to hold risky assets:

$$E(R_i^c) = \gamma_M \text{cov}(R_i^c, R_M^c) + \sum_{j=1}^L \delta_j \text{cov}(R_i^c, R_j^c), \quad (3)$$

where  $R_M^c$  is the excess return on the world market portfolio, and  $\gamma_M$  and  $\delta_j$  are, respectively, the prices of market and currency risk, defined (in time varying form) in equations (4) and (5).

It has become standard over the last decade to test the (International) CAPM conditionally. This can be done by specifying conditional first and second moments, or time varying  $\gamma_M$  and  $\delta_j$ , or both. In this way, the econometrician can better capture the fact that investors update their expectations as new information becomes available (see, for instance, surveys of Bollerslev, Chou and Kroner 1992; and Pagan 1996). The *conditionally* estimated version of Adler and Dumas' model is:

$$E(R_{i,t+1}^c | \Psi_t) = \gamma_{M,t} \text{cov}(R_{i,t+1}^c, R_{M,t+1}^c | \Psi_t) + \sum_{j=1}^L \delta_{j,t} \text{cov}(R_{i,t+1}^c, R_{j,t+1}^c | \Psi_t), \quad (3')$$

where  $E(\cdot | \Psi_t)$  and  $\text{cov}(\cdot | \Psi_t)$  are, respectively, the market expectation and covariance of excess returns, both conditional on the current information set  $\Psi_t$ . All returns are in excess of the riskless interest rate denominated

in the measurement currency. Therefore,  $R_{i,t+1}^c$  is the nominal excess rate of return on asset  $i$  (from time  $t$  to  $t + 1$ ). Finally,  $R_{j,t+1}^c$  is the nominal excess return on short-term currency Eurocurrency deposits of non-reference country  $j$ . (Once translated into dollars, deposit returns become stochastic due to changes in the exchange rate.)  $\gamma_{M,t}$  and  $\delta_{j,t}$  are defined as follows:

$$\gamma_{M,t} = \rho_t, \quad (4)$$

$$\delta_{j,t} = \rho_t(1/\rho_{j,t} - 1)W_{j,t}/W_t, \quad (5)$$

where  $\rho_t$  is the weighted average of the Arrow-Pratt coefficients of relative risk aversion of all national investors ( $\rho_{j,t}$ ) from countries  $j = 1, \dots, L + 1$ , i.e.  $1/\rho_t = \sum_{j=1}^{L+1}(W_{j,t}/\rho_{j,t})/W_t$ , and  $W_t$  is the sum of each country's wealth  $W_{j,t}$ , that is  $W_t = \sum_{j=1}^{L+1} W_{j,t}$ . The coefficient  $\gamma_{M,t}$  can be interpreted as the world price of market risk, because  $cov(R_{i,t+1}^c, R_{M,t+1}^c | \Psi_t)$  measures the non-diversifiable market risk exposure. By the same token, since  $cov(R_{i,t+1}^c, R_{j,t+1}^c | \Psi_t)$  measures the exposure of asset  $i$  to the currency risk of country  $j$ , the coefficients  $\delta_{j,t}$  can be interpreted as the price of exchange rate risk of that country. Note that in the classic CAPM no exchange risk hedging premium appears, so the International CAPM can be reduced to a closed-economy CAPM by imposing  $\delta_{j,t} = 0$ ,  $j = 1, \dots, L$ . The model postulates that the price of market risk is positive (since investors are assumed to be risk averse), and equal across all (integrated) markets in the world. There are no restrictions on the sign of the prices of currency risk, however. To better see this, let USD be the reference currency. A negative price of, say, Yen risk would mean that investors are willing to forego some expected return on an asset  $i$  if the asset return is positively correlated with the USD/Yen exchange rate. This is perfectly plausible if investors' consumption basket is tilted towards goods priced in Yen. They would then dislike Yen appreciation, would seek a hedge in the form of an asset which yields positive expected return (measured in USD) when Yen appreciates (i.e. positive covariance), and would be willing to sacrifice some return on the asset if such a hedge asset is found. Interestingly, De Santis, Gerard and Hillion (1998) find precisely this result for the US dollar, when choosing DEM as reference currency.

Evidence for time dependence of the prices of market and/or currency risk was established by, among others, Harvey (1989, 1991), Chou, Engle and Kane (1992), Bekaert and Harvey (1995), Dumas and Solnik (1995), De Santis and Gerard (1997, 1998), De Santis, Gerard and Hillion (1998), and

Cappiello (1998). These authors, apart from Chou *et al.*, introduce time dependence in the prices of risk through a set of local and global information variables, an approach which finds support in much of the latent variables literature (for instance Hansen and Hodrick 1983; and Gibbons and Ferson 1985). Time-varying prices of risk allow for variability in the coefficients of risk aversion. Constant prices, on the other hand, would imply that risk premia change only with changing (co)variances. However, we will *not* allow the prices of risk to vary over time, for the following reason. When we estimate the International CAPM with regime switching GARCH parameters, we are confronted with a large parameter proliferation with obvious computational problems. In order to keep the estimation tractable, we are forced to assume constant prices of risk. Since we use the non-switching International CAPM as benchmark, its prices of risk are also kept constant.

Order the risky assets of the integrated world as follows: first list the  $L + 1$  risky stock market indices (one for each country). Next, list the  $L$  risky Eurocurrency deposits denominated in the  $L$  non-reference currencies. Finally, add the risky world stock market portfolio as the last asset. As noted, all returns are measured in the reference currency; we therefore drop the superscript  $c$  from now on. Vectorize the excess returns and the conditional covariances. When assuming rational expectations and constant prices of risk, the econometric model which emerges from equation (3') is, in the more compact matrix notation:

$$\mathbf{R}_{t+1} = \gamma_M \mathbf{h}_{2(L+1),t+1} + \sum_{j=1}^L \delta_j \mathbf{h}_{L+1+j,t+1} + \boldsymbol{\varepsilon}_{t+1}, \quad \boldsymbol{\varepsilon}_{t+1} \mid \boldsymbol{\Psi}_t \sim N(0, \mathbf{H}_{t+1}), \quad (6)$$

where  $\mathbf{R}_{t+1}$  is the  $2(L+1) \times 1$  vector of excess returns of the risky assets. We assume that  $\boldsymbol{\varepsilon}_{t+1}$ , the vector of residuals, is conditionally normally distributed with zero mean and covariance matrix  $\mathbf{H}_{t+1}$ . The latter has dimension  $2(L+1) \times 2(L+1)$ , and is modelled as a GARCH(1,1). The  $2(L+1) \times 1$  vector  $\mathbf{h}_{2(L+1),t+1}$ , the last  $(2(L+1)th)$  column of  $\mathbf{H}_{t+1}$ , contains the conditional covariances between the  $2(L+1)$  excess asset returns and the excess return on the world market portfolio. The  $2(L+1) \times 1$  vector  $\mathbf{h}_{L+1+j,t+1}$ , the  $(L+1+j)th$  column of  $\mathbf{H}_{t+1}$ , contains the conditional covariances between the  $2(L+1)$  excess asset returns and the excess Eurocurrency deposit return for non-reference currency  $j$  ( $j = 1, 2, \dots, L$ ).

## 2.2 GARCH Specification

We assume that the conditional covariance matrix follows the GARCH(1,1) model proposed by Baba, Engle, Kraft, and Kroner (BEKK, see Engle and Kroner 1995):

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \mathbf{A}'\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{A} + \mathbf{B}'\mathbf{H}_t\mathbf{B}, \quad (7)$$

where  $\mathbf{C}$ ,  $\mathbf{A}$  and  $\mathbf{B}$  are  $2(L+1) \times 2(L+1)$  matrices of parameters.  $\mathbf{C}\mathbf{C}'$  is the Cholesky decomposition of a symmetric positive definite constant term matrix. As such,  $\mathbf{C}$ , which is a lower triangular matrix, contains only  $[2(L+1) \times (2(L+1) + 1)]/2$  parameters. The  $2(L+1) \times 1$  vector of error terms  $\boldsymbol{\varepsilon}_t$  is the difference between the realized and expected returns. To reduce the number of parameters, it is often postulated that  $\mathbf{A}$  and  $\mathbf{B}$  are diagonal, at the cost of assuming away any spillovers in volatility across markets, because the (co)variances in  $\mathbf{H}_{t+1}$  will depend only on their own past squared residuals and autoregressive components<sup>3</sup>. In this case, equation (7) reduces to:

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \mathbf{a}\mathbf{a}' \odot \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \mathbf{b}\mathbf{b}' \odot \mathbf{H}_t, \quad (8)$$

where the  $2(L+1) \times 1$  vectors  $\mathbf{a}$  and  $\mathbf{b}$  are the diagonal elements of the matrices  $\mathbf{A}$  and  $\mathbf{B}$ , and  $\odot$  is the Hadamard product operator (element by element matrix multiplication). This model has “only”  $n(n+5)/2$  parameters, where  $n = 2(L+1)$ .

An important issue to address is the number of assets and countries to include in the model. Since the International CAPM postulates that in an integrated world the excess return on the stock market in country  $i$  includes the sum of risk premia demanded by all domestic and foreign investors, we should ideally include country  $i$  plus *all* countries integrated with it. Clearly, this is not feasible. Therefore, one commonly includes only high-market-cap countries, plus the relevant proxies for the exchange rates and the world stock market portfolio. Even with this simplification, the system of equations expands rapidly. For instance, with 3 countries ( $L = 2$ ) plus two exchange rates and the world stock market portfolio, one gets  $2(L+1) = 6$  assets, i.e. 6 mean equations for returns: 3 national stock market indices, 2 risky Eurocurrency deposits, and the stock market index for the world. When using the diagonal BEKK GARCH specification one then gets 33 parameters to estimate, plus 3 prices of risk (one for the market and two for the currency risk). This is already a significant number of parameters to estimate. There are two ways to ease this problem. First, as pointed out by De Santis, Gerard and Hillion (1998), one is not obliged to include all the stock market equations; only the assets which are pricing

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<sup>3</sup>Assuming diagonal  $\mathbf{A}$  and  $\mathbf{B}$  is admittedly restrictive for high-frequency data; previous research has documented cross-market dependencies in conditional volatility at high frequencies (see, for example, Hamao, Masulis and Ng 1990; and Chan, Karolyi and Stulz 1992). However, by introducing regime switching GARCH specification, we will argue that some of the volatility spillovers are effectively reinstated (see section 2.3).



factors (world stock market index and all the Eurocurrency deposits) must be included in full, since they are part of a system of equations. Leaving out some stock markets, however, reduces the power of the model estimation. Second, one could choose a more parsimonious but less general (diagonal) GARCH representation, like the one proposed by Ding and Engle (1994):

$$\mathbf{H}_{t+1} = \mathbf{H}_0 \odot (\boldsymbol{\nu}' - \mathbf{a}\mathbf{a}' - \mathbf{b}\mathbf{b}') + \mathbf{a}\mathbf{a}' \odot \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \mathbf{b}\mathbf{b}' \odot \mathbf{H}_t, \quad (9)$$

where  $\mathbf{H}_0$  is the unconditional covariance matrix of returns,  $\boldsymbol{\nu}$  is a vector of ones, and the vectors  $\mathbf{a}$  and  $\mathbf{b}$  are defined above. The advantage of this representation is that it requires the estimation of only the elements of the vectors  $\mathbf{a}$  and  $\mathbf{b}$ , reducing the number of GARCH parameters to  $2n$ , where  $n = 2(L+1)$ . Referring to our example above, the Ding-Engle representation requires estimation of only 12 parameters. The disadvantage, however, is that this GARCH specification is not guaranteed to be positive definite (for proof, see Ding and Engle 1994), contrary to the BEKK model (which is positive definite under mild conditions, even for diagonal representations; see Engle and Kroner 1995). Moreover, it assumes that the underlying conditional covariance process is stationary.

We have chosen the BEKK GARCH representation for the following reasons. First, it is more general than the Ding-Engle, since it does not rest on the assumption of covariance stationarity. Second, and more importantly, when estimating our regime switching version of the GARCH-M process, the Ding-Engle specification turned out not to be positive definite at every point in time, and consequently the likelihood function could not be evaluated.

### 2.3 Regime Switching GARCH

As discussed in section 1.1, GARCH parameters can turn out to be unstable over time (Diebold 1986; and Lamoureux and Lastrapes 1990). Such time variation, if not accounted for, can cause spurious persistence and poor forecasting performance (Hamilton and Susmel 1994). When estimating the International CAPM, we therefore allow the parameters of the GARCH-M specification to switch according to the filter proposed by Hamilton (1988, 1989, 1990, 1994). Hamilton's model has been widely applied both in finance and macroeconomics, since it is able to capture, in a stochastic fashion, nonlinearities in time series.

In our analysis we postulate that regime switches are governed by a latent Markov variable,  $s_t$ , which defines two states of the GARCH parameters. The BEKK GARCH representation (equation (8)) then becomes:

$$\mathbf{H}_{t+1}(s_t, \dots, s_0) = \mathbf{C}\mathbf{C}'(s_t) + \mathbf{a}\mathbf{a}'(s_t) \odot \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'(s_{t-1}, \dots, s_0) + \mathbf{b}\mathbf{b}'(s_t) \odot \mathbf{H}_t(s_{t-1}, \dots, s_0). \quad (10)$$

The system of equations we therefore estimate is based on equation (6), augmented with regime switching:

$$\mathbf{R}_{t+1} = \gamma_M \mathbf{h}_{2(L+1), t+1}(s_t, \dots, s_0) + \sum_{j=1}^L \delta_j \mathbf{h}_{L+1+j, t+1}(s_t, \dots, s_0) + \boldsymbol{\varepsilon}_{t+1}(s_t, \dots, s_0), \quad (11)$$

$$\boldsymbol{\varepsilon}_{t+1} \mid \boldsymbol{\Psi}_t \sim N[0, \mathbf{H}_{t+1}(s_t, \dots, s_0)].$$

All symbols have been defined in section 2.1.

Notice first that the conditional covariance matrix  $\mathbf{H}_{t+1}$  now depends on the entire history of state variables. At each point in time the conditional covariance matrix  $\mathbf{H}_{t+1}$  will depend on the current regime, *and* on all past regimes (through the past squared error terms and past conditional covariance). The reason is that any GARCH process, univariate or multivariate and regardless of the mean equation, “remembers” the entire past, due to the inclusion of the past conditional (co)variances. From an econometric point of view this is problematic, since the resulting likelihood function needed for Maximum Likelihood (ML) estimation has to account for all possible paths of the latent variable. If one postulates the existence of  $K$  regimes, one must, in period  $t$ , account for  $K^t$  possible paths, or terms, in the likelihood function<sup>4</sup>. This makes estimation virtually impossible for large sample sizes, underlying the view of Hamilton and Susmel (1994) and Cai (1994) that regime switching GARCH models are essentially intractable and impossible to estimate. To get around this problem, when implementing a regime switch, researchers have usually dealt with ARCH specifications, rather than GARCH, despite the fact that GARCH(1,1) is better able to describe stock market volatility than is an ARCH<sup>5</sup>.

Gray (1996), who pioneered regime switching in the parameters of a univariate GARCH model, suggested a way out of this problem. At each

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<sup>4</sup>To see this, assume there are only two regimes. For each time period the log-likelihood function is composed of two densities, one for each regime, weighted with the conditional probability of being in the respective regime. Each of these probabilities will depend on the regime in the previous period, which in turn depends on the regime two periods back, and so on, back to the beginning of the time series.

<sup>5</sup>Indeed a GARCH(1,1) can be seen as a parsimonious representation of an infinite order ARCH.

point in time, the dependence on the regime path is effectively removed by aggregating over all possible regimes, and the conditional variance depends only on current regime. Dueker (1997) also implemented regime switching in a univariate GARCH. In the spirit of Kim (1994), he devised a scheme in which it is necessary to keep track of only the two most recent state dependent variances, which are averaged out<sup>6</sup>.

Moving, like we do, to regime switching in a *multivariate GARCH-M* is therefore highly non-trivial and has until now only been attempted bivariate for an International CAPM without currency risk (Bekaert and Harvey 1995), and for a domestic CAPM (Santos 1999). Our six-variate framework implies a significant increase in the number of unknown parameters. Our contribution thus goes beyond previous research. It differs from the work of Bekaert and Harvey in that we have more assets, more risk premia, and a GARCH(1,1) specification as opposed to an ARCH(3). Compared with Santos, we again have more assets and risk premia, and more importantly, we do not impose a global switch on the covariance matrix; rather we allow for individual switches of GARCH parameters. On the other hand Santos specifies three regimes, while we only assume two.

Before explaining how we solve the problem of proliferating number of terms in the likelihood function, we first discuss the number of state variables and regimes, and describe how the parameters are made to switch.

We assume that regime switches are governed by one single latent Markov variable. This implies that switches in volatilities and covariances will occur simultaneously in all international markets. This synchrony might seem restrictive. However, it can be argued that it reintroduces spillover effects in volatilities across stock markets, as evidenced by unconditional correlations of squared returns (table 1 panel F), but lost when adopting a diagonal GARCH representation. Nevertheless, we deem it unrealistic that both stock and money markets switch at the same time, with the exception of stock and money markets belonging to the same country/region (see table 1 panel F). We address this issue by letting only stock markets switch, with the additional advantage of saving parameters to estimate. The disadvantage, of course, is that we will not capture any associated switches in the money markets. A possible way out of this *impasse* could be to equip each market with its own state variable, a proposition which, however, would render the estimation non-feasible on anything but a supercomputer.

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<sup>6</sup>The univariate regime switching GARCH of Dueker (1997) was introduced in four different specifications: i) a regime switching constant term in the GARCH; ii) regime switching in the ARCH parameter of the GARCH; iii) regime switching in the student- $t$  degrees-of-freedom parameter; and iv) as for iii) but also with a switching scale factor for the conditional variance.

Furthermore, we assume the existence of two regimes only, in order to avoid excessive parameter proliferation. More specifically,  $s_t \in \{1, 2\}$  and the Markov transition probability matrix is defined as follows<sup>7</sup>:

$$\begin{aligned} P(s_t = 1|s_{t-1} = 1) &= p, & P(s_t = 1|s_{t-1} = 2) &= 1 - q, \\ P(s_t = 2|s_{t-1} = 1) &= 1 - p, & P(s_t = 2|s_{t-1} = 2) &= q. \end{aligned} \quad (12)$$

These Markov transition probabilities are used to calculate filter and *ex ante* conditional probabilities, which serve to infer in which regime the state variable is at period  $t$ , and to forecast next period's regime, respectively. They are computed by iterating on the following two equations:

$$P(s_t = 1|\Psi_t; \theta) = \frac{P(s_t = 1|\Psi_{t-1}; \theta) f(\mathbf{R}_{t+1}|s_t = 1, \Psi_t; \theta)}{\sum_{k=1}^2 P(s_t = k|\Psi_{t-1}; \theta) f(\mathbf{R}_{t+1}|s_t = k, \Psi_t; \theta)}, \quad (13)$$

$$P(s_t = 2|\Psi_t; \theta) = 1 - P(s_t = 1|\Psi_t; \theta), \quad (13')$$

$$P(s_{t+1} = 1|\Psi_t; \theta) = p P(s_t = 1|\Psi_t; \theta) + (1 - q) P(s_t = 2|\Psi_t; \theta), \quad (14)$$

$$P(s_{t+1} = 2|\Psi_t; \theta) = 1 - P(s_{t+1} = 1|\Psi_t; \theta), \quad (14')$$

where equations (13)-(13') represent the filter probabilities for the two regimes, and equations (14)-(14') are the *ex ante* probabilities.  $f(\mathbf{R}_{t+1}|s_t = k, \Psi_t; \theta)$ ,  $k = 1, 2$ , are the densities in the two regimes.  $\theta$  is the vector of unknown parameters.

GARCH parameters are allowed to switch by multiplying the elements of the  $\mathbf{C}$  matrix and the  $\mathbf{a}$  and  $\mathbf{b}$  vectors with individual switching coefficients. These coefficients are normalized to one in the second regime. Therefore it is sufficient to define only three (instead of six) vectors of switching coefficients,  $\mathbf{g}\mathbf{c}$ ,  $\mathbf{g}\mathbf{a}$ , and  $\mathbf{g}\mathbf{b}$ , multiplying  $\mathbf{C}$ ,  $\mathbf{a}$  and  $\mathbf{b}$ , respectively. For example,  $\mathbf{g}\mathbf{a} = [ga_1 \ ga_2 \ ga_3 \ 1 \ 1 \ ga_6]'$ , where the fourth and fifth elements are set equal to one, since the two money markets are assumed not to switch. Thus, in regime 1,  $\mathbf{C}$ ,  $\mathbf{a}$  and  $\mathbf{b}$  are modified as follows:

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<sup>7</sup>Note that in the regime switching model of Hamilton (1988, 1989, 1990, 1994) the transition probabilities are constant. However, there is an increasing interest for time varying transition probabilities in the literature (Ghysels 1993; Diebold, Lee and Weinbach 1994; Filardo 1994; Durland and McCurdy 1994; Bekaert and Harvey 1995; Gray 1996; Whitelaw 1998; Boudoukh, Richardson, Smith, and Whitelaw 1999; Perez-Quiros and Timmerman 1999; and Dahlquist and Gray 2000). While intuitively appealing, we do not adopt such an approach, since it would aggravate an already serious parameter proliferation.

$$\mathbf{C}^{(1)} \equiv \mathbf{g}\mathbf{c} \mathbf{g}\mathbf{c}' \odot \mathbf{C},$$

$$\mathbf{a}^{(1)} \equiv \mathbf{g}\mathbf{a} \odot \mathbf{a}, \tag{15}$$

$$\mathbf{b}^{(1)} \equiv \mathbf{g}\mathbf{b} \odot \mathbf{b},$$

where the superscript refers to the first regime. As mentioned,  $\mathbf{C}^{(2)} \equiv \mathbf{C}$ ,  $\mathbf{a}^{(2)} \equiv \mathbf{a}$ , and  $\mathbf{b}^{(2)} \equiv \mathbf{b}$ . An alternative approach would have been to define two different  $\mathbf{C}$  matrices as well as  $\mathbf{a}$  and  $\mathbf{b}$  vectors. The advantage of our approach is that it saves parameters when specifying  $\mathbf{C}^{(1)}$ : we only need to estimate the  $n(n+1)/2 = 21$  parameters in  $\mathbf{C}$ , where  $n = 2(L+1) = 6$ , plus the four elements of  $\mathbf{g}\mathbf{c}$ . Instead, had we used the other approach, we would have had to estimate 12 distinct parameters in  $\mathbf{C}^{(1)}$  plus 21 parameters in  $\mathbf{C}^{(2)}$ <sup>8</sup>. As for  $\mathbf{a}$  and  $\mathbf{b}$ , the two approaches are equivalent.

We now turn to the methodology we adopt to handle the proliferation of ML terms. Bekaert and Harvey (1995), for each period, average out the error terms coming from different regimes, using the *ex ante* probabilities as weights. Santos (1999) follows a similar approach for a switching GARCH-M. We, too, opt for this technique. Hence, for each observation, we average out the elements of the conditional covariance matrix in each regime:

$$\widehat{\mathbf{H}}_t = P(s_t = 1 | \Psi_{t-1}; \boldsymbol{\theta}) \mathbf{H}_t^{(1)} + P(s_t = 2 | \Psi_{t-1}; \boldsymbol{\theta}) \mathbf{H}_t^{(2)}, \tag{16}$$

where  $\widehat{\mathbf{H}}_t$  is state independent. Since elements of  $\widehat{\mathbf{H}}_t$  enter the mean equations, the resulting vector of error terms  $\widehat{\boldsymbol{\varepsilon}}_t$  will be state independent as well. Both  $\widehat{\mathbf{H}}_t$  and  $\widehat{\boldsymbol{\varepsilon}}_t$  are subsequently used in the GARCH equation (8), which has two possible regimes:

$$\mathbf{H}_{t+1}^{(1)} = \mathbf{C}^{(1)}\mathbf{C}^{(1)'} + \mathbf{a}^{(1)}\mathbf{a}^{(1)'} \odot \widehat{\boldsymbol{\varepsilon}}_t\widehat{\boldsymbol{\varepsilon}}_t' + \mathbf{b}^{(1)}\mathbf{b}^{(1)'} \odot \widehat{\mathbf{H}}_t, \tag{17}$$

$$\mathbf{H}_{t+1}^{(2)} = \mathbf{C}\mathbf{C}' + \mathbf{a}\mathbf{a}' \odot \widehat{\boldsymbol{\varepsilon}}_t\widehat{\boldsymbol{\varepsilon}}_t' + \mathbf{b}\mathbf{b}' \odot \widehat{\mathbf{H}}_t. \tag{17'}$$

We estimate three different combinations of equation (17): First, we allow only the constant matrix  $\mathbf{C}$  to switch (giving a total of 42 unknown parameters, which include the transition probabilities and the vector  $\mathbf{g}\mathbf{c}$ ). Next, we let only the vectors  $\mathbf{a}$  and  $\mathbf{b}$  switch (46 unknown parameters). Finally, we make both the matrix  $\mathbf{C}$  and the two vectors  $\mathbf{a}$  and  $\mathbf{b}$  switching (50 unknown parameters).

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<sup>8</sup>The reason why  $\mathbf{C}^{(1)}$  contains fewer distinct parameters than  $\mathbf{C}^{(2)}$  is that the money markets are not allowed to switch, i.e.  $c_{4i}^{(1)} = c_{4i}^{(2)}$  for  $i = 1, \dots, 4$ , and  $c_{5i}^{(1)} = c_{5i}^{(2)}$  for  $i = 1, \dots, 5$ .

## 3 Data

This section describes the data set used for the analysis. It provides descriptive statistics and diagnostic tests (normality, autocorrelations, and cross correlations) for excess asset returns and their squares, and for the instrumental variables used in the specification tests. We show that the time series used exhibit some of the features captured by GARCH, namely autocorrelation in squared returns, or volatility clustering. Moreover, the unconditional distributions are non-normal, showing excess kurtosis and skewness, which GARCH processes to some extent can model. It is also well known that asset returns are characterized by jumps, which contribute to excess kurtosis (Bates 1996; Bekaert, Erb, Harvey and Viskanta 1998; and Das and Upal 1998). Also regime switching models should therefore accommodate fat tails. A combination of the two econometric tools should potentially provide an even better fit of the fat tails. Finally, there is significant evidence for unconditional contemporaneous volatility spillovers between assets, indicating that a diagonal BEKK representation is not fully appropriate. However, our synchronous regime switching of the GARCH parameters should restore some of the spillover effects.

All observations are from the last trading day of the week, and cover the period from February 7 1986 to December 31 1998, for a total sample size of 674.

Although our data sample only covers a period of 13 years, it still allows us to capture the stock market crash in 1987, the financial turmoil in Europe in 1992, and the recent Asian/Russian/Latin American financial crisis, and it gives us a sufficient number of observations. Moreover, we must stress that the International CAPM is a model for integrated financial markets, where cross-border investments are assumed frictionless and investors show no home bias. Our use of weekly data represents a novelty in the estimation of an International CAPM with currency risk premia included. It allows us to concentrate our study on a recent time period characterized by growing financial liberalization and integration. Moreover, GARCH models are designed to capture volatility clustering, which might be more predominant in weekly than in monthly data (Bollerslev, Chou and Kroner 1992).

### 3.1 Asset Data

We use (excess) stock market value-weighted return indices cum dividends for USA, Europe, Japan, and the world. The first three stock markets represent around 95% of the world market capitalization (end of 1998). The

index for Europe is a weighted basket of national indices<sup>9</sup>. The total return indices, available in US dollars, have been obtained from Financial Times Actuaries (FT/S&P World series) at a weekly frequency.

Bid rates on Eurocurrency deposits for the above mentioned countries or regions, needed to calculate the (excess) returns on currency holdings, have been obtained from Bank of International Settlements (BIS), at weekly frequencies. The returns on Eurocurrency deposits are translated into US dollars, using the exchange rate implicit in the FT/S&P total return indices. Exchange rates are defined as USD/Yen and USD/European basket currency. We point out that a Eurocurrency deposit rate for “Europe”, associated to the basket index *FT/S&P World Europe*, does not exist. The closest we get is the EuroECU deposit rate. This inconsistency is, however, of minor significance, since only five small countries represented in the *FT/S&P World Europe* basket are missing in the ECU currency basket<sup>10</sup>.

All returns are in excess of the one-month Eurodollar deposit rate (used as risk free asset), USD being our reference currency.

Descriptive statistics for the asset data are given in table 1. Panel 1A shows that all distributions exhibit skewness and leptokurtosis at 1% significance level (with the exception of EuroECU deposits), a clear sign of non-normality. This is confirmed by the Jarque-Bera normality test.

Panel 1B gives autocorrelations for lags between 1 and 6. With two exceptions, there are no significant autocorrelations, neither at 1% or 5% significance level. As expected, excess returns show no autocorrelation, as indicated by the Ljung-Box statistics. Therefore, we do not need to add autoregressive (AR) terms in the mean equations.

Panel 1C presents unconditional correlations between the six assets, which are seen to be significant at the 1% level (except for 5% level for USA-EuroYen). It is interesting to notice, though, that the US and Japanese stock markets show a correlation of only 21%, while correlations between Europe and USA (and Europe-Japan) do not exceed 50%. These “low” correlations, generally smaller than those of *national* assets, suggest that international diversification is beneficial. Finally, panel 1C also reveals an almost 50% correlation between returns in national stock and money markets, and between EuroECU and EuroYen deposit returns.

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<sup>9</sup>The basket, called *FT/S&P World Europe*, comprises the following national stock market indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and UK.

<sup>10</sup>These five countries are: Austria, Finland, Norway, Sweden and Switzerland.

**Table 1**  
**Descriptive statistics of weekly excess total returns on stock market indices and Eurocurrency deposits**

*Panel 1A: Distributional statistics of excess returns*

	<b>USA</b>	<b>Japan</b>	<b>Europe</b>	<b>Yen</b>	<b>ECU</b>	<b>World</b>
<b>Mean</b>	0.222	0.057	0.214	0.048	0.063	0.165
<b>Min.</b>	-12.785	-12.541	-16.313	-6.252	-6.468	-13.917
<b>Max.</b>	7.458	18.034	11.261	14.894	6.487	9.364
<b>St.dev.</b>	2.000	3.410	2.170	1.697	1.354	1.891
<b>Skew.</b>	-0.6230**	0.540**	-0.280**	1.387**	-0.006	-0.102**
<b>Kurt.</b>	7.458**	5.612**	11.072**	12.586**	5.328**	9.822**
<b>J-B</b>	601.696**	224.407**	1838.599**	2796.609**	152.256**	1308.224**

\*\* denotes 1% significance level.

USA, Japan, Europe and World represent the US, Japanese, European and world stock market indices, while Yen and ECU are the EuroYen and EuroECU deposit returns.

Mean, min., max. and st.dev. (standard deviation) are in %.

The significance level for skewness (skew.) and excess kurtosis (kurt.) is based on test statistics developed by D'Agostino, Belanger and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as  $\chi_m^2$  with  $m = 2$  degrees of freedom.

*Panel 1B: Autocorrelations of excess returns*

	<b>USA</b>	<b>Japan</b>	<b>Europe</b>	<b>Yen</b>	<b>ECU</b>	<b>World</b>
$\rho_1$	-0.027	-0.021	0.025	-0.015	0.022	0.021
$\rho_2$	0.061	0.083*	0.065	0.045	0.039	0.054
$\rho_3$	-0.019	0.020	0.001	0.042	0.053	0.015
$\rho_4$	-0.030	0.018	-0.034	0.050	-0.050	-0.017
$\rho_5$	-0.008	0.046	-0.037	0.049	0.043	0.028
$\rho_6$	0.077*	-0.022	0.012	-0.031	-0.013	0.027
<b>L-B<sub>36</sub></b>	50.171	49.324	32.919	46.445	26.292	34.860

\* denotes 5% significance level.

$\rho_k$  is the autocorrelation function at lag  $k$ .

The Ljung-Box $_m$  (L-B $_m$ ) statistics tests the null hypothesis that all autocorrelation coefficients are simultaneously equal to zero up to  $m$  lags; it is asymptotically distributed as  $\chi_m^2$ . We have chosen  $m = 36$  for which the critical values at 95% and 99% confidence levels are 50.998 and 58.619, respectively.



Panel 1C: Unconditional correlations of excess returns

	USA	Japan	Europe	Yen	ECU	World
USA	1.000	0.210**	0.489**	-0.088*	-0.117**	0.715**
Japan		1.000	0.448**	0.567**	0.258**	0.731**
Europe			1.000	0.218**	0.422**	0.798**
Yen				1.000	0.483**	0.275**
ECU					1.000	0.227**
World						1.000

\* and \*\* denote 5% and 1% significance levels, respectively.

In Panel 1D we report cross correlations between the excess returns on the US stock market index and those on the other five assets, for 6 lags and leads, as well as contemporaneous. Among the 60 non-contemporaneous correlations in this panel, only 6 are significant. There are, of course, five more tables of cross correlations, not shown here, between the remaining pairs of assets. This gives a total of 180 non-contemporaneous correlations, of which only 14 turn out to be significant (including the 6 already mentioned). This shows that leading or lagging spillovers in excess returns are rare, i.e. the markets do not tend to lead others. Estimation is therefore facilitated, since we do not need to model non-contemporaneous spillovers in the mean equations. At *daily* frequency, these spillovers do in fact become relevant, and they have been modelled by Chan, Karolyi and Stulz (1992), and Hamao, Masulis and Ng (1990).

While excess returns in levels have little autocorrelation, excess *squared* returns exhibit more significant autocorrelation (Panel 1E). As indicated by the Ljung-Box test statistics, four of the markets show autocorrelation at 1% significance level, one at 5%, and only the world stock market index is not significantly autocorrelated. Thus, the serial correlation detected here argues in favor of a GARCH representation.

In Panel 1F we show that there is significant contemporaneous cross correlation between squared excess returns, with the exception of EuroYen deposits (only correlated with the national stock market). This suggests that a diagonal GARCH parametrization may not be appropriate for our sample. However, as mentioned, we argue that the synchronous regime switching which we introduce in stock markets' second moments effectively restores, to some extent, the contemporaneous volatility spillovers.

Panel 1D: Cross correlations of excess returns between US stock market index and the other five assets

	Japan	Europe	Yen	ECU	World
$t_{-6}$	0.047	0.039	-0.087*	-0.004	0.063
$t_{-5}$	-0.046	-0.039	0.008	-0.041	-0.031
$t_{-4}$	-0.018	-0.011	-0.065	-0.013	-0.041
$t_{-3}$	0.016	0.023	-0.041	0.027	0.016
$t_{-2}$	0.023	0.006	0.003	-0.008	0.029
$t_{-1}$	-0.013	0.014	-0.048	0.002	0.027
$t_0$	0.210**	0.489**	-0.088*	-0.117**	0.715**
$t_1$	0.044	0.110**	0.046	-0.048	0.056
$t_2$	0.028	0.100**	0.002	-0.079*	0.067
$t_3$	-0.027	-0.005	0.033	-0.069	-0.029
$t_4$	-0.019	0.026	0.043	-0.025	0.001
$t_5$	0.075*	0.059	-0.089*	0.013	0.068
$t_6$	-0.049	-0.002	0.048	0.000	0.019

\* and \*\* denote 5% and 1% significance levels, respectively.

$t_{-m}$  represents the cross correlation at lag  $m$ , while  $t_m$  is the cross correlation at lead  $m$ .

Panel 1E: Autocorrelations of squared excess returns

	USA	Japan	Europe	Yen	ECU	World
$\rho_1$	0.245**	0.131**	-0.003	0.055	0.019	0.041
$\rho_2$	0.083*	0.082*	0.055	0.021	0.013	0.021
$\rho_3$	0.002	0.054	-0.040	-0.007	0.039	0.000
$\rho_4$	0.051	0.082*	0.092*	0.019	0.233**	0.016
$\rho_5$	0.072	0.076*	0.100**	0.166**	0.062	0.123**
$\rho_6$	0.140**	0.012	0.035	-0.002	-0.022	0.066
<b>L-B<sub>36</sub></b>	117.030**	68.951**	57.082*	61.758**	109.750**	33.747

\* and \*\* denote 5% and 1% significance levels, respectively.

$\rho_k$  is the autocorrelation function at lag  $k$ .

The Ljung-Box <sub>$m$</sub>  (L-B <sub>$m$</sub> ) statistics tests the null hypothesis that all autocorrelation coefficients are simultaneously equal to zero up to  $m$  lags; it is asymptotically distributed as  $\chi_m^2$ . We have chosen  $m = 36$ , for which the critical values at 95% and 99% confidence levels are 50.998 and 58.619, respectively.

Panel 1F: Unconditional correlations of squared excess returns

	USA	Japan	Europe	Yen	ECU	World
USA	1.000	0.289**	0.688**	0.003	0.079*	0.756**
Japan		1.000	0.337**	0.280**	0.131**	0.563**
Europe			1.000	0.002	0.341**	0.851**
Yen				1.000	0.096*	0.009
ECU					1.000	0.135**
World						1.000

\* and \*\* denote 5% and 1% significance levels, respectively.

As for *leading and lagging* cross correlations of squared excess returns, we find 44 significant values out of a total of 180 pairs up to 6 leads/lags. Panel 1G shows only cross correlations with the US stock market, where we find 14 significant figures (excluding contemporaneous ones). All in all, one sees that there is no strong need for modelling non-contemporaneous cross correlations.

### 3.2 Instrument Data

For robustness tests, we use instrumental variables to check if there is predictability left in the estimated residuals. The instruments we consider are commonly used prediction variables for asset returns (Harvey 1991; Bekaert and Hodrick 1992; Campbell and Hamao 1992; Ferson and Harvey 1993, 1994; Harvey, Solnik and Zhou 1994; and Bekaert and Harvey 1995). These information variables can be grouped into global and local ones. The three globals are: i) The change in the 1-month Eurodollar deposit rate ( $\Delta\text{EUSD}$ ; source: BIS); ii) The change in the US default spread ( $\Delta\text{USDS}$ ), defined as Moody's Baa minus Aaa long-term bond rate (source: Federal Reserve); and iii) The change in the US term structure spread ( $\Delta\text{USTS}$ ), measured by the yield to maturity on the 10 year Treasury bond in excess of the 3-month T-bill rate (source: Federal Reserve). The locals are the change in the real 1-month EuroYen (EuroECU) deposit rate in excess of the real 1-month Eurodollar rate ( $\Delta\text{RYD}$  and  $\Delta\text{RECUD}$ ), where real interest rates are computed by the Fisher formula using national inflation rates (source: International Financial Statistics)<sup>11</sup>.

<sup>11</sup>The inflation rate for Europe has been calculated as a weighted average of national inflation rates, where the weights are chosen to be those of the ECU basket currencies. Hence, countries not represented in the ECU basket do not contribute to the European inflation rate.

Panel 1G: Cross correlations of squared excess returns between US stock market index and the other five assets

	<b>Japan</b>	<b>Europe</b>	<b>Yen</b>	<b>ECU</b>	<b>World</b>
$t_{-6}$	0.059	0.116**	0.069	-0.040	0.075*
$t_{-5}$	-0.009	0.101**	0.097*	-0.007	0.106**
$t_{-4}$	0.034	0.039	0.042	0.037	0.048
$t_{-3}$	-0.021	-0.013	0.036	-0.018	0.007
$t_{-2}$	0.000	0.006	0.019	-0.031	0.049
$t_{-1}$	0.030	0.007	0.080*	-0.015	0.220**
$t_0$	0.289**	0.688**	0.003	0.079*	0.756**
$t_1$	0.072	0.214**	0.188**	-0.004	0.041
$t_2$	-0.021	0.083*	0.014	0.019	0.009
$t_3$	0.006	0.006	0.008	-0.008	-0.004
$t_4$	0.028	0.066	0.048	0.049	0.007
$t_5$	0.113**	0.093*	-0.003	-0.005	0.072
$t_6$	0.000	0.082*	0.052	-0.040	0.139**

\* and \*\* denote 5% and 1% significance levels, respectively.

$t_{-m}$  represents the cross correlation at lag  $m$ , while  $t_m$  is the cross correlation at lead  $m$ .

Descriptive statistics are provided in table 2. Although the instruments show some autocorrelation, it is generally small. More importantly, most of the instruments are uncorrelated, which means that they do not provide redundant information. However, the exception is  $\Delta\text{EUSD}$ , which is correlated with both local information variables. As we will point out in sections 4.1.1 and 4.2.1, we solve this problem by using either  $\Delta\text{EUSD}$  or a local information variable. Finally, the local instruments are mutually correlated, but, again, they will not appear in the same specification tests.

## 4 Estimation

In this section we present empirical results for the International CAPM. We estimate the model with constant prices of risk, in order to make a fair comparison between the non-switching and switching specifications. The latter is very difficult to estimate if prices of risk are made time varying: adding extra parameters and instrumental variables in this highly non-linear and complex model renders estimation intractable. Section 4.1 gives results obtained with the traditional estimation method, i.e. multivariate GARCH-M *without* regime switching parameters. In section 4.2, regime switching is introduced. For the non-switching model, which is our benchmark, we obtain

**Table 2**  
**Descriptive statistics for instruments in first differences**

*Panel 2A: Distributional statistics for instruments*

	$\Delta\text{EUSD}$	$\Delta\text{USDS}$	$\Delta\text{USTS}$	$\Delta\text{RYD}$	$\Delta\text{RECUD}$
<b>Mean</b>	-0.004	-0.001	-0.003	-0.009	-0.004
<b>Min.</b>	-1.180	-0.230	-0.590	-1.682	-1.358
<b>Max.</b>	1.070	0.170	0.800	1.953	2.647
<b>St.dev.</b>	0.173	0.042	0.140	0.312	0.299

\* and \*\* denote 5% and 1% significance levels, respectively.

$\Delta\text{EUSD}$  is the change in the 1-month Eurodollar deposit rate.  $\Delta\text{USDS}$  is the change in the US default spread, defined as Moody's Baa minus Aaa long-term bond rate.  $\Delta\text{USTS}$  is the change in the US term structure spread, measured by the yield to maturity on the 10 year Treasury bond in excess of the 3-month T-bill rate.  $\Delta\text{RYD}$  is the change in the real 1-month EuroYen deposit rate in excess of the real 1-month Eurodollar rate.  $\Delta\text{RECUD}$  is the change in the real 1-month EuroECU deposit rate in excess of the real 1-month Eurodollar rate.

Mean, min., max. and st.dev. (standard deviation) are in %.

*Panel 2B: Autocorrelations for instruments*

	$\Delta\text{EUSD}$	$\Delta\text{USDS}$	$\Delta\text{USTS}$	$\Delta\text{RYD}$	$\Delta\text{RECUD}$
$\rho_1$	-0.045	-0.140**	-0.041	-0.100**	-0.063
$\rho_2$	0.013	-0.039	0.011	-0.018	-0.018
$\rho_3$	0.004	0.034	-0.053	-0.050	-0.146**
$\rho_4$	-0.023	-0.013	0.059	0.060	0.071
$\rho_5$	-0.142**	0.024	0.050	-0.035	-0.076*
$\rho_6$	0.015	-0.061	-0.025	-0.019	0.054
<b>L-B<sub>36</sub></b>	55.460*	52.737*	49.765	45.734	59.858**

\* and \*\* denote 5% and 1% significance levels, respectively.

The acronyms of the variables are explained in Panel 2A.

$\rho_k$  is the autocorrelation function at lag  $k$ .

The Ljung-Box $_m$  (L-B $_m$ ) statistics tests the null hypothesis that all autocorrelation coefficients are simultaneously equal to zero up to  $m$  lags; it is asymptotically distributed as  $\chi_m^2$ . We have chosen  $m = 36$ , for which the critical values at 95% and 99% confidence level are 50.998 and 58.619, respectively.

Panel 2C: Unconditional correlations of instruments

	$\Delta\text{EUSD}$	$\Delta\text{USDS}$	$\Delta\text{USTS}$	$\Delta\text{RYD}$	$\Delta\text{RECUD}$
$\Delta\text{EUSD}$	1.000	0.020	-0.026	-0.429**	-0.403**
$\Delta\text{USDS}$		1.000	0.034	-0.029	0.025
$\Delta\text{USTS}$			1.000	0.010	-0.029
$\Delta\text{RYD}$				1.000	0.221**
$\Delta\text{RECUD}$					1.000

\*\* denotes 1% significance level.

The acronyms of the variables are explained in Panel 2A.

a positive and significant price of market risk, but small and non-significant prices of currency risk. The risk premia are seen to react to the major financial market events in the considered sample period. As for the switching model, the price of market risk is still positive but no longer significant, while prices of currency risk remain small and not significant. Risk premia show about the same structure as those of the non-switching model. However, a more careful analysis reveals interesting differences in stock market risk exposures. First, when GARCH parameters are allowed to switch, we find that, in periods of financial turmoil, the way investors form their expectations about conditional second moments changes: More emphasis is put on the current shocks, and less on the current conditional (co)variances. Second, and more importantly, the persistence of the switching model is seen to be lower throughout crises, giving faster reaction to market shocks. This might suggest that the non-switching model has spuriously high persistence, causing it to underestimate conditional second moments at the onset, and overestimate them during the crisis. Such an outcome is in line with earlier research on different (non-CAPM type) univariate models, where switching GARCH specifications yield superior forecasts (see section 1.1).

#### 4.1 Estimation of International CAPM *without* Regime Switching GARCH Parameters

The system of mean equations we estimate is based on equation (6), with  $L = 2$ . The first three assets are the excess returns on the US, Japanese and European stock indices; the two next are excess returns on the EuroYen and EuroECU deposits; and finally the last is the excess return on the world stock market index. All returns are translated into US dollars, and are in excess of the US risk free rate. The sum is over the two non-reference currencies (Yen and ECU). As said before, we assume conditional normality for the error terms, whose covariance matrix is given by equation (8). More precisely, in scalar notation equation (6) becomes:

$$R_{i,t+1} = \gamma_M cov(R_{i,t+1}, R_{M,t+1} | \Psi_t) + \sum_{j=1}^2 \delta_j cov(R_{i,t+1}, R_{L+1+j,t+1} | \Psi_t) + \varepsilon_{t+1}, \quad (6')$$

for assets  $i = 1, \dots, 6$ . The Quasi Maximum Likelihood (QML) method of Bollerslev and Wooldridge (1992), which gives standard errors robust to departures from normality, is used to maximize the multivariate log-likelihood function. The latter is given by (see, for instance, Gouriéroux and Monfort 1995):

$$L(\boldsymbol{\theta}) = -\frac{nT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln(|\mathbf{H}_t(\boldsymbol{\theta})|) - \frac{1}{2} \sum_{t=1}^T \boldsymbol{\varepsilon}_t(\boldsymbol{\theta})' \mathbf{H}_t(\boldsymbol{\theta})^{-1} \boldsymbol{\varepsilon}_t(\boldsymbol{\theta}). \quad (18)$$

$T$  is the sample size,  $n$  is the number of assets, and  $\boldsymbol{\theta} = (\gamma_M \delta_{Yen} \delta_{ECU} c_{ij} a_i b_i)'$ ,  $i, j = 1, \dots, 6$ , is the  $(36 \times 1)$  vector of unknown parameters to be estimated. The maximization is performed using the Constrained ML module in GAUSS, employing the Newton-Raphson algorithm<sup>12</sup>.

QML gives results as reported in table 3. Notice that the price of world market risk is significant, in contrast to earlier results obtained with International CAPM incorporating currency risk and constant prices of risk (Dumas and Solnik 1995; and De Santis and Gerard 1998). The prices of currency risk, however, are not significant. Neither have other researchers found significant constant prices of currency risk. This suggests that time-varying prices might be needed. The parameters  $a_i$  and  $b_i$  are all highly significant and show the typical high persistence of GARCH models<sup>13</sup>. Still all the conditional variances and covariances are stationary, as defined by Bollerslev's (1986) decay (persistence) parameter  $\lambda_{ij} = a_i a_j + b_i b_j < 1, \forall i, j$ .

In our setting excess returns are made up of three components: the market risk premium (the first term on the right hand side of equation (6')), and the Yen and ECU currency risk premia (the second and third terms). Each component is the product of the price of risk and the respective risk exposure (measured by conditional (co)variances). In figure 1 we plot these three components, as well as the total risk premium, for each asset.

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<sup>12</sup>We also tried the BHHH (Berndt, Hall, Hall and Hausman 1974) optimization algorithm. When estimating the model without regime switching, Newton-Raphson and BHHH turned out to be equivalent. For the regime switching model, however, BHHH did not converge.

<sup>13</sup>As for the 21 elements of the constant term ( $\mathbf{C}$ ) in the GARCH specification, 12 are statistically significant.

**Table 3**  
**QML estimation results for non-switching model**

The estimated model is:

$$R_{i,t+1} = \gamma_M cov(R_{i,t+1}, R_{M,t+1} | \Psi_t) + \sum_{j=1}^2 \delta_j cov(R_{i,t+1}, R_{L+1+j,t+1} | \Psi_t) + \varepsilon_{t+1},$$

for  $i = 1, \dots, 6$ .  $\gamma_M$  is the price of world market risk, and  $\delta_{Yen}$  ( $\delta_{ECU}$ ) is the price of Yen (ECU) risk. The error terms are conditionally normally distributed, i.e.  $\varepsilon_{t+1} | \Psi_t \sim N(0, \mathbf{H}_{t+1})$ . The conditional covariance matrix follows the BEKK multivariate GARCH(1,1) specification:

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \mathbf{a}\mathbf{a}' \odot \varepsilon_t \varepsilon_t' + \mathbf{b}\mathbf{b}' \odot \mathbf{H}_t$$

*Estimated Parameters*

	<b>USA</b>	<b>Japan</b>	<b>Europe</b>	<b>Yen</b>	<b>ECU</b>	<b>World</b>
$\gamma_M$						0.0686 (0.0213)
$\delta_{Yen}$				0.0137 (0.0764)		
$\delta_{ECU}$					0.0162 (0.1176)	
$a_i$	0.2428 (0.0193)	0.2579 (0.0192)	0.2418 (0.0268)	0.2761 (0.0436)	0.2179 (0.0747)	0.2447 (0.0190)
$b_i$	0.9663 (0.0044)	0.9636 (0.0047)	0.9581 (0.0084)	0.9368 (0.0159)	0.9407 (0.0496)	0.9637 (0.0047)
Log-likelihood function						-9.3393

The 21 elements of the constant term matrix  $\mathbf{C}$  are not reported to save space. QML standard errors are shown in parentheses.



### Figure 1: Components of excess returns for non-switching model

Prices of risk are assumed to be constant. The market and currency risk premia are the first, second and third terms on the right hand side of equation (6'). The scale along the vertical axes is in units of percent.

Figure 1A: US Stock Market

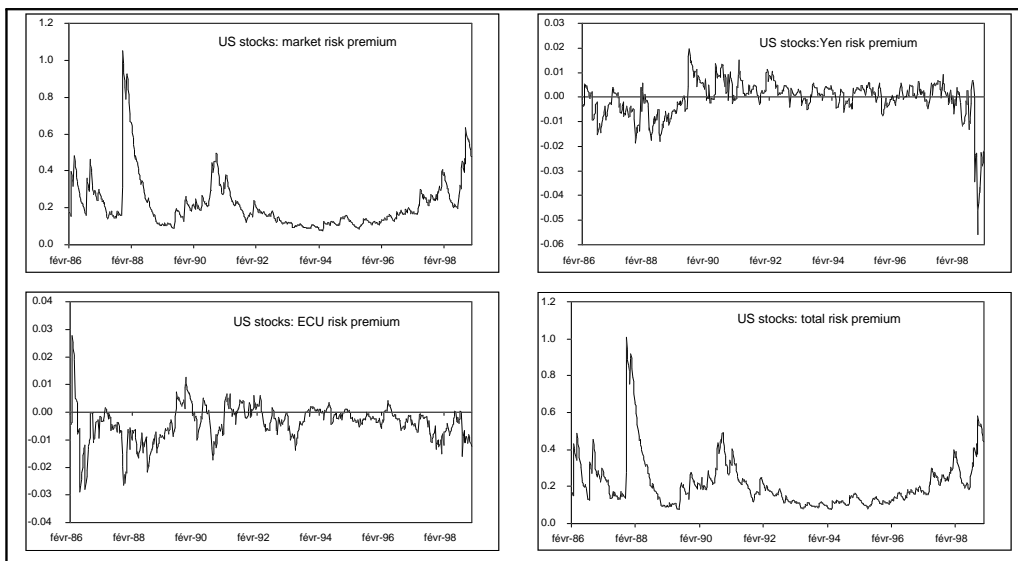


Figure 1B: Japanese Stock Market

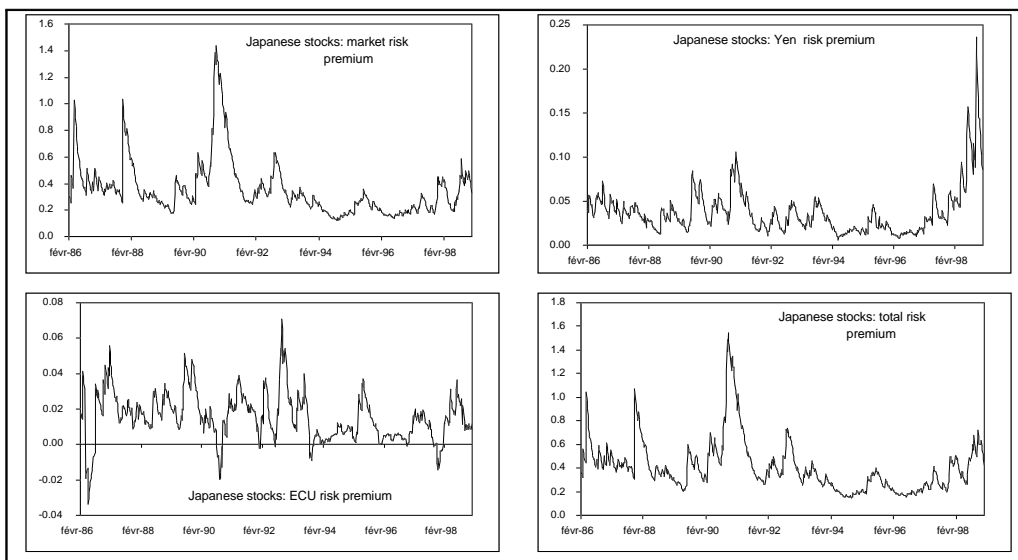


Figure 1C: European Stock Market

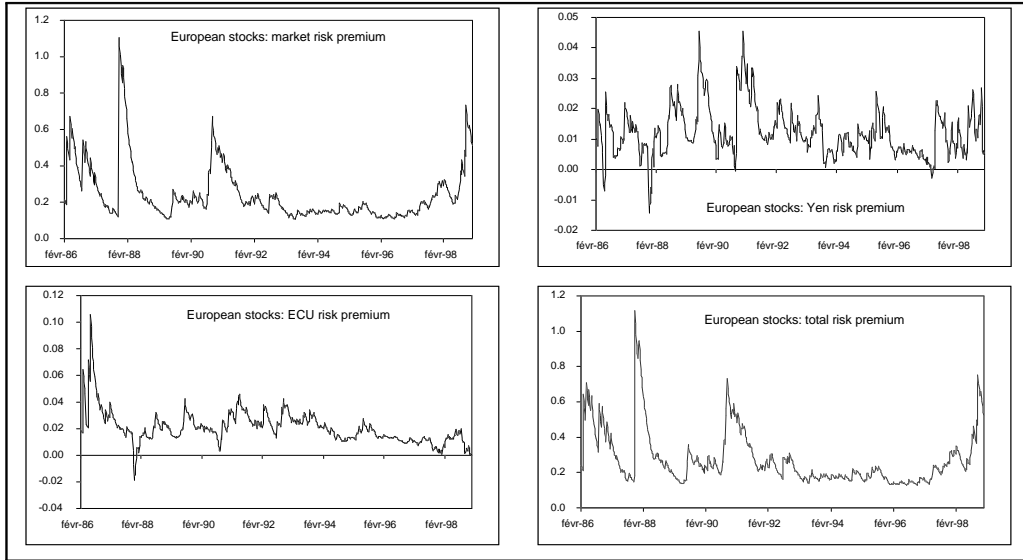


Figure 1D: EuroYen Deposit

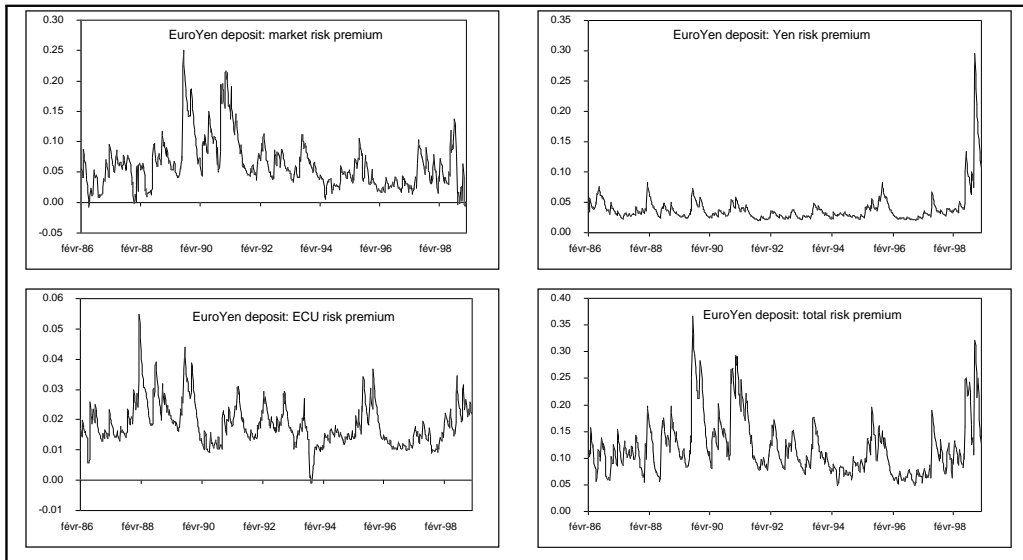


Figure 1E: EuroECU Deposit

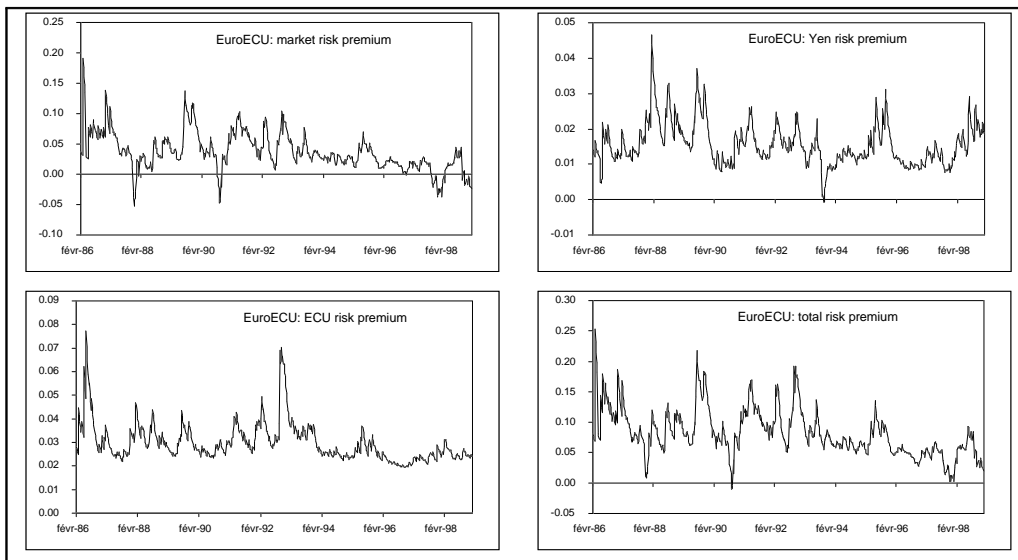
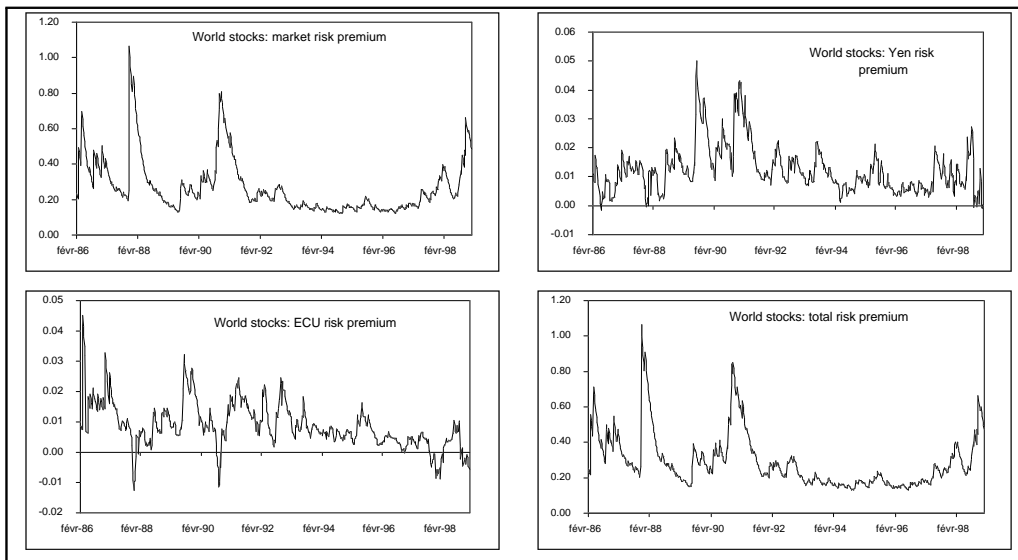


Figure 1F: World Stock Market



The US market risk premium is seen to rise substantially during the summer/autumn 1986 market turbulence, the October 1987 stock market crash, the 1990/91 Gulf War, and the 1997/98 Asian-Russian-Latin American crises, as shown in figure 1A. Moreover, since currency premia are negligible compared with the market premium, the total risk premium is almost entirely driven by the market premium. Nevertheless, the currency premia have some interesting structure: the Yen risk premium shows a sudden drop at the end of 1998, when the US stock market fell and the Yen appreciated against the US dollar. This generated a temporarily strong negative correlation (since USD/Yen increased), which US investors, demanding a positive price of Yen risk, rewarded by requiring a lower total risk premium. As for the ECU risk premium, it is seen to drop after the October 1987 stock market crash and during Gulf war, and also, modestly, during the EMS crisis of 1992/93.

The Japanese stock market (figure 1B) picks up the turbulence in summer/autumn 1986, and shows dramatic increases in market risk premium in 1987 (contagion from Wall Street), as well as during the Gulf War. Notice that the effect of this conflict is larger on the Japanese market than on the US, probably due to Japan's heavier reliance on imported oil. Also noticeable are the effects of the bursting stock market bubble in the spring of 1990, the Kobe earthquake in January 1995, and the recent Asian crisis. Moreover, there is always a positive conditional correlation between returns on Japanese stocks and EuroYen deposits, which is also seen unconditionally (see table1, panel C). The Yen risk premium reaches its maximum during the Asian crisis, when the conditional correlation increased between Japanese equity and currency markets. The same positive relationship can be observed between Japanese stocks and EuroECU deposits. The most pronounced peak occurs during the EMS crisis, due to the depreciation of the ECU against the dollar and a fall in the Japanese stock market. Finally, as for the US stock market, the total risk premium on Japanese stocks is predominantly determined by the market risk premium.

For European stocks the market risk premium reacts to the volatile equity markets of mid-1986, and surges during the 1987 crash, the Gulf War, and the Asian-Russian-Emerging Market crisis at the end of 1998 (figure 1C). Interestingly, the 1992 EMS currency crisis has a much smaller effect on European stocks. As for the Yen risk premium, it is mostly positive and small, but not negligible. ECU risk premium is also positive. The total risk premium is again dominated by the market risk.

The EuroYen risk premium (see figure 1D) is essentially determined by two equally important components: the market risk premium and the Yen risk premium, where the latter is, in fact, a conditional variance. Two major peaks appear in the market risk premium, one during the 1989 mini-crash and one during the Gulf War, due to the positive conditional correlations between Yen depreciation and the world stock market decline. The 1987 crash has, instead, comparably little effect. Concerning the Yen risk premium, the sharp spike at the end of 1998 reflects the increased conditional variance of Yen during the Asian crisis. The ECU risk premium, on the other hand, is quite small and almost always positive. Hence, there is a positive correlation between the two money markets, which can be attributed to a tendency of co-movements in both Eurodeposit rates as well as Yen and ECU exchange rates against the dollar.

Risk premia on EuroECU deposits are shown in figure 1E. The market and the ECU risk premia have similar magnitudes. The ECU risk premium is non-negligible because it is a conditional variance. The Yen risk premium mirrors the ECU risk premium on EuroYen deposits discussed above, the only difference being the proportionality factor (price of risk). It is worth noting the increase of the ECU risk premium in 1986, when there was tension within EMS with resulting interest rate increases and exchange rate realignments, and in 1992, the time of the fullblown EMS crisis.

Finally, the risk premia on the world stock market are shown in figure 1F. Not surprisingly, given that the world market portfolio is (almost) a weighted average of the three considered regions, we see the usual peaks in mid-1986, October 1987, the Gulf War, and during the emerging market crises of 1997/98. Since the risk exposure of world stocks to Yen is the same as that of the EuroYen deposit to the world market, the associated premia show identical time evolution. The same identity holds for world stocks' risk exposure to ECU and EuroECU deposits' risk exposure to the world market, commented on above. The total risk premium is almost entirely driven by the market premium.

It is worth noting that the results obtained and presented in table 3 and figure 1 are able to capture, in a remarkably clear way, the major events in international financial markets, and to significantly price the world market risk. Such clean and significant signals are typically missing in earlier work (Bekaert and Harvey 1995; De Santis and Gerard 1998; and De Santis, Gerard and Hillion 1998).

### 4.1.1 Specification Tests

In this section we analyze the standardized estimated residuals, in levels and squared. The objectives are twofold: to check whether the GARCH model is able to capture all the leptokurtosis shown by the data, as well as the autocorrelation of squared returns; and, secondly, to test if the mean equations are correctly specified.

Table 4 panel A reports distributional statistics for the normalized estimated residuals,  $\nu_{i,t} = \varepsilon_{i,t}/\sqrt{H_{ii,t}}$ , where  $i = 1, \dots, 6$ , and  $H_{ii,t}$  are the diagonal elements of the conditional covariance matrix  $\mathbf{H}_t$  (equation (8)). While the GARCH process is unable to substantially reduce the excess kurtosis of returns, except for EuroYen deposits (table 1 panel A), it captures the autocorrelation in squared returns: As seen from table 1 panel E, the Ljung-Box test statistics is significant for all assets apart from the world stock market, whereas in squared normalized residuals it is significant only for the EuroECU deposit.

The second set of tests we perform checks whether the information variables described in section 3.2 have explanatory power for the estimated residuals  $\varepsilon_{i,t}$  (not normalized). More precisely, for the US and the world we regress the residuals on a constant term and the three lagged global instruments. For Japanese/European stocks and Yen/ECU Eurodeposits, two regressions are computed: One with a constant term plus only global variables, and another where the change in the Eurodollar deposit rate is replaced by a local instrument<sup>14</sup>. Obviously, for Japanese stocks and EuroYen deposits (European stocks and EuroECU deposits) the local information variable is the change in the real 1-month EuroYen (EuroECU) deposit rate in excess of the real 1-month Eurodollar rate. Adjusted  $R^2$  from each regression are negligible, indicating that instruments have very weak explanatory power. This result is reinforced by a Wald test, which is  $\chi^2_{(m)}$  distributed,  $m$  representing the number of restrictions. More specifically, we test two null hypotheses: *i*)  $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$  and *ii*)  $H_0 : \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$ , where  $\beta_0$  is the constant term of the regression and  $\beta_i$ ,  $i = 1, \dots, 3$ , are the slope coefficients. When testing *i*) the null cannot be rejected at 95% confidence level (CL), except for EuroECU deposits, which, however, cannot be rejected at 99% CL. When testing *ii*) the null cannot be rejected at 95% CL, with the exception of Japanese stocks, which, however, cannot be rejected at 99% CL. It is therefore interesting to note that, even though we do not

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<sup>14</sup>Notice that we do not retain the change in the Eurodollar deposit rate when using local instruments, since, as documented in table 2, panel C, this information variable is significantly correlated with the local instruments.

**Table 4**  
**Diagnostics tests on normalized estimated residuals for**  
**non-switching model**

*Panel 4A: Distributional statistics for levels and squared normalized residuals*

	<b>USA</b>	<b>Japan</b>	<b>Europe</b>	<b>Yen</b>	<b>ECU</b>	<b>World</b>
<b>Mean</b> ( $\nu_{i,t}$ )	0.013	-0.128	-0.020	-0.041	-0.005	-0.065
<b>St.dev.</b> ( $\nu_{i,t}$ )	0.929	1.025	0.976	1.007	0.991	0.968
<b>Kurt.</b> ( $\nu_{i,t}$ )	5.017**	5.564**	12.576**	6.125**	4.600**	9.347**
<b>J-B</b> ( $\nu_{i,t}$ )	128.780**	191.749**	2617.487**	358.876**	74.748**	1139.307**
<b>L-B<sub>36</sub></b> ( $\nu_{i,t}$ )	49.311	35.629	24.658	46.914	25.545	34.029
<b>L-B<sub>36</sub></b> ( $\nu_{i,t}^2$ )	42.204	22.867	11.624	43.844	74.320**	13.274

\*\* denotes 1% significance level.

$\nu_{i,t}$  are the normalized estimated residuals:  $\nu_{i,t} = \varepsilon_{i,t} / \sqrt{H_{ii,t}}$ , where  $i = 1, \dots, 6$ , and  $H_{ii,t}$  are the estimated diagonal elements of the conditional covariance matrix  $\mathbf{H}_t$  (equation (8)).

USA, Japan, Europe and World represent the US, Japanese, European and world stock market indices, while Yen and ECU are the EuroYen and EuroECU deposits.

Mean and st.dev. (standard deviation) are in %.

The significance level for excess kurtosis (kurt.) is based on test statistics developed by D'Agostino, Belanger and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as  $\chi_m^2$  with  $m = 2$  degrees of freedom. The Ljung-Box <sub>$m$</sub>  (L-B <sub>$m$</sub> ) statistics tests the null hypothesis that all autocorrelation coefficients are simultaneously equal to zero up to  $m$  lags; it is asymptotically distributed as  $\chi_m^2$ . We have chosen  $m = 36$  for which the critical values at 95% and 99% confidence levels are 50.998 and 58.619, respectively.

model prices of risk as a function of these information variables, the latter turn out not to be relevant in explaining estimated residuals.

## 4.2 Estimation of International CAPM *with* Regime Switching GARCH Parameters

The model we estimate is derived from equation (11), where all but the most recent states are averaged out according to the procedure described in section 2.3. More precisely, the estimated equations in scalar notation become:

$$R_{i,t+1} = \gamma_M \text{cov}(R_{i,t+1}, R_{M,t+1} | s_t = k, \Psi_t) + \sum_{j=1}^2 \delta_j \text{cov}(R_{i,t+1}, R_{L+1+j,t+1} | s_t = k, \Psi_t) + \varepsilon_{t+1}^{(k)}, \quad (19)$$

for assets  $i = 1, \dots, 6$  and states  $k = 1, 2$ . The state dependent error terms are conditionally normally distributed, i.e.  $\varepsilon_{t+1}^{(k)} | \Psi_t \sim N(0, \mathbf{H}_{t+1}^{(k)})$ , where  $\mathbf{H}_{t+1}^{(k)}$  is given by equations (17) and (17'). As discussed in section 2.3, three different specifications for a switching  $\mathbf{H}_{t+1}^{(k)}$  have been studied. When only the constant matrix  $\mathbf{C}$  is allowed to switch, there is no evidence for two distinct regimes since one of the two Markov transition probabilities approaches one and, as a consequence, the associated smoothed probability is almost always very close to one. Next, making both the matrix  $\mathbf{C}$  and the two vectors  $\mathbf{a}$  and  $\mathbf{b}$  switching, the program fails to converge. Finally, we let only the vectors  $\mathbf{a}$  and  $\mathbf{b}$  switch, where the transitions are governed by a common latent variable. Results for this specification are presented below.

Again, we use QML to estimate the  $46 \times 1$  vector of unknown parameters  $\boldsymbol{\theta} = (\gamma_M \ \delta_{Yen} \ \delta_{ECU} \ c_{ij} \ a_i \ b_i \ ga_i \ gb_i \ p \ q)'$ ,  $i, j = 1, \dots, 6$ . The multivariate log-likelihood function is a weighted average of state dependent densities, with weights given by the *ex ante* probabilities specified in equations (14)-(14'):

$$L(\boldsymbol{\theta}) = \sum_{t=1}^T \ln \left[ \sum_{k=1}^2 P(s_{t-1} = k | \Psi_{t-2}; \boldsymbol{\theta}) f(\mathbf{R}_t | s_{t-1} = k, \Psi_{t-1}; \boldsymbol{\theta}) \right], \quad (20)$$

where  $T$  is the sample size.  $f(\cdot | \cdot)$  is the conditional density in regime  $k$ :

$$f(\mathbf{R}_t | s_{t-1} = k, \Psi_{t-1}; \boldsymbol{\theta}) =$$



Panel 4B: Regression of estimated residuals on instruments

	USA	Japan	Europe	Yen	ECU	World
$\overline{R}_{global}^2$	0.002	0.006	-0.001	-0.002	0.011	0.001
$\overline{R}_{local+global}^2$		0.006	-0.002	-0.000	0.007	
$\chi_{(3),global}^2$	1.795 (0.616)	3.469 (0.325)	1.583 (0.663)	1.247 (0.742)	8.325 (0.040)	1.427 (0.699)
$\chi_{(3),local+global}^2$		3.518 (0.318)	1.117 (0.773)	2.201 (0.532)	6.788 (0.079)	
$\chi_{(4),global}^2$	1.864 (0.761)	11.832 (0.019)	2.367 (0.669)	2.614 (0.624)	8.378 (0.079)	4.307 (0.366)
$\chi_{(4),local+global}^2$		11.837 (0.019)	1.812 (0.770)	4.205 (0.379)	7.227 (0.124)	

$\overline{R}_{global}^2$  is the adjusted  $R^2$  for the regression of the estimated residuals on the three global instruments.  $\overline{R}_{local+global}^2$  is the adjusted  $R^2$  for the regression of the estimated residuals on two global instruments and the relevant local information variable. The globals are: i) The change in the 1-month Eurodollar deposit rate; ii) The change in the US default spread, defined as Moody's Baa minus Aaa long-term bond rate; iii) The change in the US term structure spread, measured by the yield to maturity on the 10 year Treasury bond in excess of the 3-month T-bill rate. The locals are the change in the real 1-month EuroYen (EuroECU) deposit rate in excess of the real 1-month Eurodollar rate, where real interest rates are computed by the Fisher formula using local inflation rates.

$\chi_{(3),global}^2$  is the Wald test statistics to test the null hypothesis that the slope regression coefficients are jointly equal to zero, in the regression containing only global instruments.  $\chi_{(3),local+global}^2$  is the corresponding test statistics when one of the globals is replaced by a local information variable.  $\chi_{(4),global}^2$  and  $\chi_{(4),local+global}^2$  are the Wald test statistics to test the null that all coefficients, including the constant term, are jointly equal to zero. Probability values are shown in parentheses.

$$\frac{1}{(2\pi)^{n/2} \sqrt{|\mathbf{H}_t^{(k)}(\boldsymbol{\theta})|}} \exp \left[ -\frac{1}{2} \boldsymbol{\varepsilon}_t^{(k)}(\boldsymbol{\theta})' \mathbf{H}_t^{(k)}(\boldsymbol{\theta})^{-1} \boldsymbol{\varepsilon}_t^{(k)}(\boldsymbol{\theta}) \right], \quad (21)$$

where  $n$  equals the number of assets. As before, the maximization is carried out with the Constrained ML module in GAUSS, using the Newton-Raphson algorithm.

QML estimation results are shown in table 5. Contrary to the results for non-switching GARCH, the price of market risk is no longer significant at any conventional confidence level. Moreover, currency risks are, again, not significantly priced. As for the GARCH parameters, all elements of vectors  $\mathbf{a}$  and  $\mathbf{b}$  are significant. However, since  $b_2$  hits an upper limit imposed as a necessary (but not sufficient) condition for GARCH stationarity, its standard error is not computed. Attempts to overcome this problem are discussed in Appendix A1<sup>15</sup>. The elements of the switching coefficient vectors  $\mathbf{ga}$  and  $\mathbf{gb}$ , as well as the Markov transition probabilities  $p$  and  $q$ , are also significant, pointing towards the existence of two distinct regimes<sup>16</sup>.

Following Hamilton (1989), we can compute the two regimes' expected duration in units of weeks,  $ED^{(k)}$ , conditional on being in either of them:

$$ED^{(1)} = \sum_{i=1}^{\infty} i p^{i-1} (1-p) = \frac{1}{1-p} = 9.5, \quad (22)$$

$$ED^{(2)} = \sum_{i=1}^{\infty} i q^{i-1} (1-q) = \frac{1}{1-q} = 39.8. \quad (22')$$

This shows that the first regime is much less persistent than the second, which lasts about 9 months.

To interpret the results for the two states, we calculate *smoothed* probabilities of being in regime 1 (see figure 2), through the following algorithm due to Kim (1993):

$$P(s_t = 1 | \boldsymbol{\Psi}_T; \boldsymbol{\theta}) = P(s_t = 1 | \boldsymbol{\Psi}_t; \boldsymbol{\theta}) \left[ p \frac{P(s_{t+1} = 1 | \boldsymbol{\Psi}_T; \boldsymbol{\theta})}{P(s_{t+1} = 1 | \boldsymbol{\Psi}_t; \boldsymbol{\theta})} + (1-p) \frac{P(s_{t+1} = 2 | \boldsymbol{\Psi}_T; \boldsymbol{\theta})}{P(s_{t+1} = 2 | \boldsymbol{\Psi}_t; \boldsymbol{\theta})} \right]. \quad (23)$$

<sup>15</sup>If we release the upper limit, the program does not converge.

<sup>16</sup>A more precise way of assessing the existence of two distinct regimes would be to carry out the Hansen (1992, 1996) test, which overcomes the problem of the nuisance parameters  $p$  and  $q$ , unidentified under the null hypothesis of no regime switching. Hansen's test, however, is highly cumbersome and will, therefore, not be performed here.

The advantage of using smoothed over *ex ante* or filter probabilities is that the former are calculated iteratively from the last observation backwards, thus taking into account the entire data sample. Figure 2 shows that the probability of being in regime 1 rises during periods of financial turmoil. The most prominent peak occurs in October 1987, when stock markets crashed worldwide. The mini-crash of October 1989, and the initial burst of the Japanese stock market bubble (winter/spring 1990) are also captured, although less dramatically. More spectacular is the rise during the Gulf war in the winter 1991 and throughout the ERM crises in 1992-1993. Furthermore, also the Asian crisis is reflected by the sharp increases in the smoothed probabilities, starting in 1997 and lasting to the end of the sample. There are three additional major peaks (end of 1998, spring 1994, and summer 1995) which appear to be related to big changes in the EuroYen and EuroECU deposit rates.

In regime 1 the GARCH parameters in vector  $\mathbf{b}$  are multiplied by coefficients  $gb_i$  less than unity, whereas, conversely, the elements of  $\mathbf{a}$  are scaled up by coefficients  $ga_i$  larger than unity. Such a shift in the expected (co)variance process, away from GARCH and towards ARCH, occurs in conjunction with the turbulent periods mentioned above. The phenomenon leads to an appealing and intuitive conclusion: When forming their expectations in periods of crises, investors attach more weight to the current shocks than to the current conditional (co)variance. In other words, their memory of the past fades, while concern about the present is enhanced. This “crisis mentality” has a short expected duration, as seen from equation (22).

Risk premia are computed using elements of the averaged conditional covariance matrix described in equation (16). In particular, the market risk premium is given by  $\gamma_M \widehat{cov}(R_{i,t+1}, R_{M,t+1} | \Psi_t)$ , whereas the Yen and ECU risk premia are defined by  $\delta_j \widehat{cov}(R_{i,t+1}, R_{j,t+1} | \Psi_t)$ , for  $i = 1, \dots, 6$ , and  $j = Yen, ECU$ , where  $\widehat{cov}(\cdot | \Psi_t)$  are elements of the conditional covariance matrix  $\widehat{\mathbf{H}}_t$  in equation (16). As shown in figure 3, these premia appear, on first sight, to be quite similar to those obtained in the non-switching model, the main difference being the scale as determined by the prices of risk. This is not surprising because the risk exposures do, and should, show broadly the same structure: They should rise during turmoil and fall in calm periods. For this reason, figure 3 only reports the *total* risk premium for each asset, and not the three components. Interesting differences can only be detected by subtracting conditional risk exposures of the switching model from those of the non-switching one (see figure 4). The figures pertaining to differences in market risk exposures of stock markets show that

**Table 5**  
**QML estimation results for regime switching model**

The estimated model is:

$$R_{i,t+1} = \gamma_M \text{cov}(R_{i,t+1}, R_{M,t+1} | s_t = k, \Psi_t) + \sum_{j=1}^2 \delta_j \text{cov}(R_{i,t+1}, R_{L+1+j,t+1} | s_t = k, \Psi_t) + \varepsilon_{t+1}^{(k)},$$

for assets  $i = 1, \dots, 6$ , and states  $k = 1, 2$ . The state dependent error terms are conditionally normally distributed, i.e.  $\varepsilon_{t+1}^{(k)} | \Psi_t \sim N(0, \mathbf{H}_{t+1}^{(k)})$ . The state dependent conditional covariance matrix follows the BEKK multivariate GARCH(1,1) specification:

$$\begin{aligned} \mathbf{H}_{t+1}^{(1)} &= \mathbf{C}^{(1)} \mathbf{C}^{(1)'} + \mathbf{a}^{(1)} \mathbf{a}^{(1)'} \odot \widehat{\boldsymbol{\varepsilon}}_t \widehat{\boldsymbol{\varepsilon}}_t' + \mathbf{b}^{(1)} \mathbf{b}^{(1)'} \odot \widehat{\mathbf{H}}_t, \\ \mathbf{H}_{t+1}^{(2)} &= \mathbf{C} \mathbf{C}' + \mathbf{a} \mathbf{a}' \odot \widehat{\boldsymbol{\varepsilon}}_t \widehat{\boldsymbol{\varepsilon}}_t' + \mathbf{b} \mathbf{b}' \odot \widehat{\mathbf{H}}_t, \end{aligned}$$

where  $\mathbf{a}^{(1)} \equiv \mathbf{g} \mathbf{a} \odot \mathbf{a}$ ,  $\mathbf{b}^{(1)} \equiv \mathbf{g} \mathbf{b} \odot \mathbf{b}$ , while  $\widehat{\boldsymbol{\varepsilon}}_t$  and  $\widehat{\mathbf{H}}_t$  are the weighted averages of the state dependent error terms and conditional covariance matrices, respectively, the weights being the ex ante probabilities.

*Estimated Parameters*

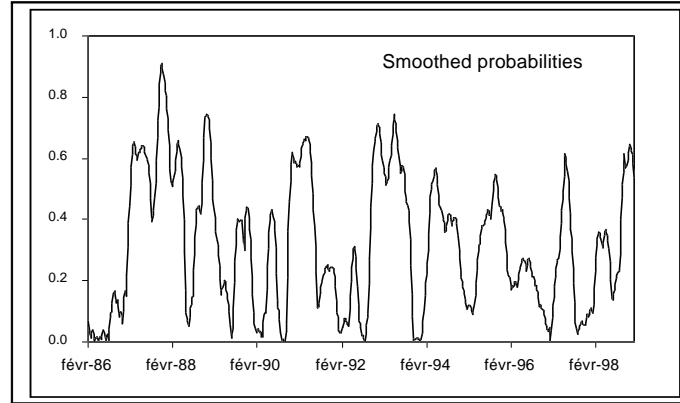
	USA	Japan	Europe	Yen	ECU	World
$\gamma_M$						0.0522 (0.0685)
$\delta_{Yen}$				0.0107 (0.0434)		
$\delta_{ECU}$					0.0059 (0.3863)	
$a_i$	0.2110 (0.0376)	0.2136 (0.0324)	0.1840 (0.0439)	0.2731 (0.0504)	0.2065 (0.0948)	0.2047 (0.0306)
$b_i$	0.9857 (0.0033)	0.9999 (-)	0.9883 (0.0073)	0.9480 (0.0249)	0.9555 (0.0606)	0.9917 (0.0033)
$ga_i$	1.4638 (0.3680)	1.5350 (0.3373)	1.9148 (0.5776)			1.6295 (0.3521)
$gb_i$	0.9216 (0.0161)	0.8680 (0.0199)	0.9095 (0.0208)			0.8959 (0.0158)

$p$	0.8942 (0.0369)	$q$	0.9749 (0.0202)	Log-likelihood function	-9.2247
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The 21 elements of the constant term matrix  $\mathbf{C}$  are not reported to save space. QML standard errors are shown in parentheses.

**Figure 2: Smoothed probability of being in regime 1**

In regime 1 the GARCH parameter vectors  $\mathbf{a}$  and  $\mathbf{b}$  are Hadamard multiplied by the vectors of switching coefficients  $\mathbf{ga}$  and  $\mathbf{gb}$ .

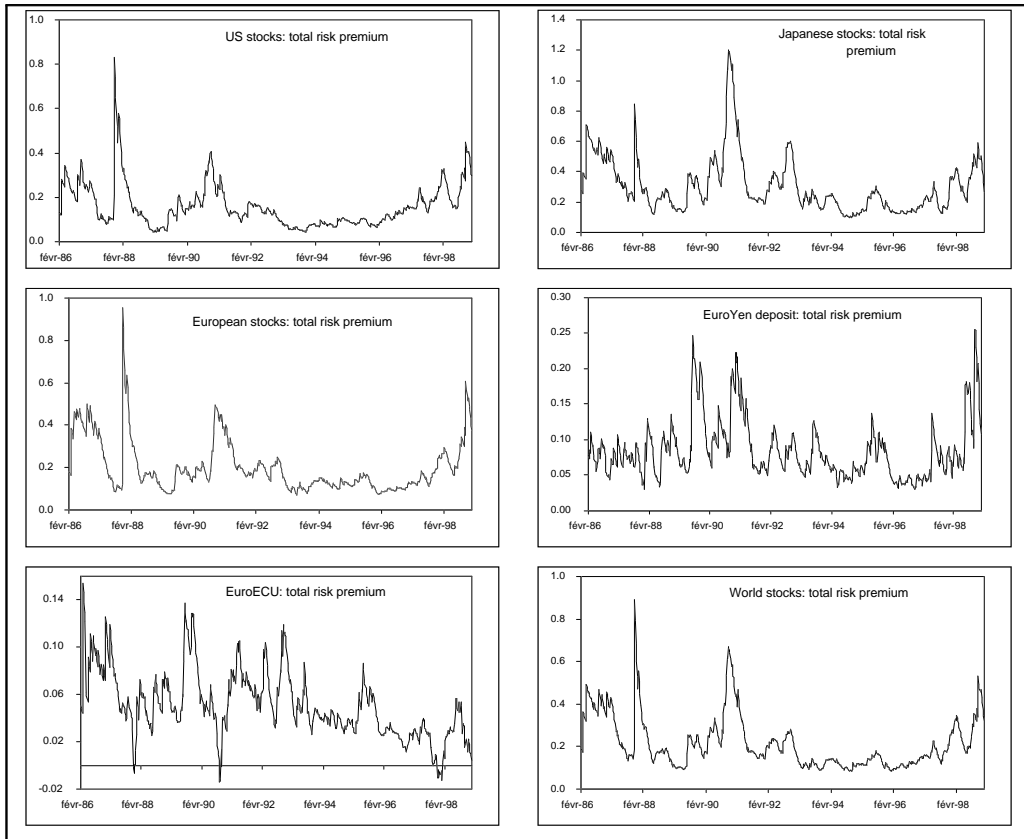


the non-switching GARCH, when compared to the switching, yields a lower conditional (co)variance at the time when it goes from a low (or normal) to a high state, and, similarly, a higher conditional (co)variance when it passes from a high to a low (or normal) state. This apparent under- and over-estimation may be attributed to spuriously high persistence of the non-switching model. Interestingly, this is precisely the behavior foreseen by the researchers who have advocated the combination of regime switching and (G)ARCH (see section 1). Take, for instance, the difference in market risk exposure for the US stock market. At the time of the October 1987 stock market crash, the difference abruptly becomes negative, and then positive shortly after. A similar feature is seen when Kuwait was invaded by Iraq in summer 1990, as well as during the ERM crisis in 1992-93, and throughout the Asian/emerging market crises in 1987-98. The Japanese, European, and world stock markets exhibit the same pattern, at the same dates. As for the money markets, the corresponding figures are much harder to interpret due to the fact that these markets are not allowed to switch. Therefore, any spurious persistence cannot be corrected, and the above mentioned pattern does not appear. For this reason, we do not show the relevant figures for money markets and currency risk exposures.

Our findings thus provide evidence that, at the onset of and throughout financial turmoils, the non-switching model has higher persistence than the switching one, suggesting that the persistence of the former model might be spuriously high. To investigate this further, we define, for the switching

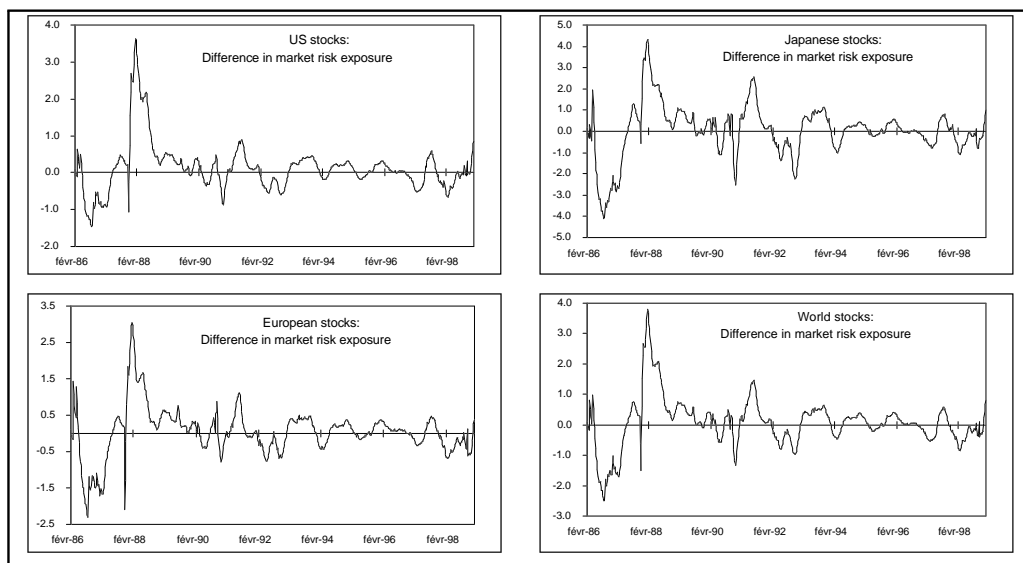
**Figure 3: Total risk premia for regime switching model**

Prices of risk are assumed to be constant. The total risk premium of asset  $i$  ( $i = 1, \dots, 6$ ) is the sum of the market and currency premia:  
 $\gamma_M \widehat{cov}(R_{i,t+1}, R_{M,t+1} | \Psi_t)$  and  $\delta_j \widehat{cov}(R_{i,t+1}, R_{j,t+1} | \Psi_t)$ , for  $j = Yen, ECU$ .  
 The scale along the vertical axes is in units of percent.



**Figure 4: Differences between conditional risk exposures for stock markets estimated with non-switching and switching GARCH parameters**

The risk exposures are the conditional (co)variances between returns on the specified market and the world stock market. The scale along the vertical axes is in units of risk exposure.



GARCH, a time varying decay parameter,  $\bar{\lambda}_{i6,t}$ , for the market risk exposures of the four stock markets:

$$\bar{\lambda}_{i6,t} = sp_t^{(1)} \left( a_i^{(1)} a_6^{(1)} + b_i^{(1)} b_6^{(1)} \right) + \left( 1 - sp_t^{(1)} \right) \left( a_i^{(2)} a_6^{(2)} + b_i^{(2)} b_6^{(2)} \right), \quad (24)$$

for  $i = 1, 2, 3, 6$ , where  $sp_t^{(1)}$  is the smoothed probability of being in regime 1 at time  $t$  (equation (23)).  $\bar{\lambda}_{i6,t}$  is analogous to the constant decay parameter  $\lambda_{i6}$  computed for the non-switching model (see section 4.1). Figure 5 shows that  $\bar{\lambda}_{i6,t}$  is lower than  $\lambda_{i6}$  in periods of crises. As already discussed above, this is due to the smaller weight given to the GARCH terms<sup>17</sup>. In more quiet periods, the reverse is true, indicating that the non-switching model only captures the *average* of two different decay parameters. Moreover, the average across time of all 21 decay parameters of the switching model,  $T^{-1} \sum_{t=1}^T \bar{\lambda}_{ij,t}$ ,  $i, j = 1, \dots, 6$ , is below one, which means that the probability weighted process is stationary.

#### 4.2.1 Specification Tests

As in section 4.1.1, diagnostic tests are carried out on standardized averaged residuals,  $\hat{v}_{i,t} = \hat{\varepsilon}_{i,t} / \sqrt{\hat{H}_{ii,t}}$ , where  $i = 1, \dots, 6$ , and  $\hat{H}_{ii,t}$  are the diagonal elements of the conditional covariance matrix  $\hat{\mathbf{H}}_t$  (equation (16)), in levels and squared. In spite of the use of GARCH and regime switching, normalized residuals still show excess kurtosis and non-normality, as indicated by the Jarque-Bera test statistics. However, the serial dependence of squared excess returns (see table 1, panel E) is captured, again except for EuroECU deposits.

We also regress (non-standardized) residuals,  $\hat{\varepsilon}_{i,t}$ , on the chosen set of lagged global and local instruments. As before, we test if the residuals for the US and world stock market are explained by a constant term plus three global information variables. For the Japanese/European stock markets and Yen/ECU Eurodeposits, an additional regression is carried out, where the change in the Eurodollar deposit rate is replaced by the relevant local instrument. The adjusted  $R^2$  are very small for all assets, again indicating very low explanatory power of the instruments. Wald tests of two null hypotheses corroborate this result, the nulls being: *i*)  $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$  and *ii*)  $H_0 : \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$ , where  $\beta_0$  is the constant term of

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<sup>17</sup>Even though, at the same time, the weight of the ARCH increases, this is not enough to compensate for the drop in the GARCH parameters.

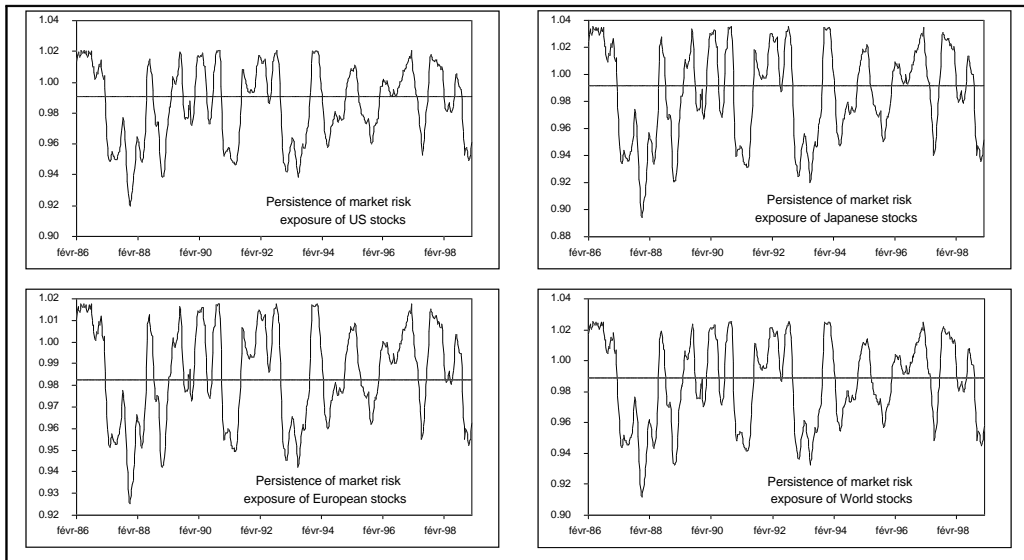


**Figure 5: Probability weighted persistence of the (co)variances between stock markets and the world stock market**

The curved lines show the time varying persistence of the market risk exposures of the stock markets, calculated as

$$\bar{\lambda}_{i6,t} = sp_t^{(1)} \left( a_i^{(1)} a_6^{(1)} + b_i^{(1)} b_6^{(1)} \right) + \left( 1 - sp_t^{(1)} \right) \left( a_i^{(2)} a_6^{(2)} + b_i^{(2)} b_6^{(2)} \right),$$

for  $i = 1, 2, 3, 6$ , where  $sp_t^{(1)}$  is the smoothed probability of being in regime 1 at time  $t$ . The horizontal lines are the corresponding constant persistences  $\lambda_{i6}$ , calculated with the non-switching model.



**Table 6**  
**Diagnostics tests on normalized estimated residuals for regime switching model**

*Panel 6A: Distributional statistics for levels and squared residuals*

	USA	Japan	Europe	Yen	ECU	World
<b>Mean</b> ( $\hat{\nu}_{i,t}$ )	0.040	-0.104	0.013	-0.019	0.017	-0.029
<b>St.dev.</b> ( $\hat{\nu}_{i,t}$ )	0.945	1.030	0.980	1.012	0.997	0.982
<b>Kurt.</b> ( $\hat{\nu}_{i,t}$ )	4.767**	4.895**	12.408**	5.672**	4.534**	9.253**
<b>J-B</b> ( $\hat{\nu}_{i,t}$ )	103.067**	102.343**	2530.581**	276.736**	68.733**	1116.311**
<b>L-B<sub>36</sub></b> ( $\hat{\nu}_{i,t}$ )	49.153	31.157	25.858	47.342	24.767	35.123
<b>L-B<sub>36</sub></b> ( $\hat{\nu}_{i,t}^2$ )	36.816	22.713	10.352	43.962	68.269**	9.773

\*\* denotes 1% significance level.

$\hat{\nu}_{i,t}$  is the normalized estimated residuals:  $\hat{\nu}_{i,t} = \hat{\varepsilon}_{i,t} / \sqrt{\hat{H}_{ii,t}}$ , where  $i = 1, \dots, 6$ , and  $\hat{H}_{ii,t}$  are the estimated diagonal elements of the conditional covariance matrix  $\hat{\mathbf{H}}_t$  (equation (16)).

USA, Japan, Europe and World represent the US, Japanese, European and world stock market indices, while Yen and ECU are the EuroYen and EuroECU deposits.

Mean and st.dev. (standard deviation) are in %.

The significance level for excess kurtosis (kurt.) is based on test statistics developed by D'Agostino, Belanger and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as  $\chi_m^2$  with  $m = 2$  degrees of freedom. The Ljung-Box <sub>$m$</sub>  (L-B <sub>$m$</sub> ) statistics tests the null hypothesis that all autocorrelation coefficients are simultaneously equal to zero up to  $m$  lags; it is asymptotically distributed as  $\chi_m^2$ . We have chosen  $m = 36$  for which the critical values at 95% and 99% confidence levels are 50.998 and 58.619, respectively.

the regression and  $\beta_i$ ,  $i = 1, \dots, 3$ , are the slope coefficients. When testing  $i$ ), the null is not rejected at 95% confidence level (CL), with the exception of EuroECU deposits, which, nevertheless, cannot be rejected at 99% CL. When testing  $ii$ ), the null is not rejected at 95% CL, except for Japanese stocks, which, however, cannot be rejected at 99% CL. These results are overall very similar to those of the non-switching model, and point towards well-specified mean equations.

## 5 Practical Applications

As already shown by, for instance, De Santis, Gerard, and Hillion (1998), estimation of a conditional International CAPM is of direct interest to investors facing global asset allocation decisions. The lifelong utility maximization which leads to (International) CAPM produces optimal weights which indicate how wealth should be allocated among different assets. These weights are functions of lagged covariance matrices, prices of risk, and expected excess returns. We have estimated these three terms, for highly integrated financial markets, using Adler and Dumas' (1983) International CAPM. Moreover, since a conditional version of this International CAPM has been applied, both first and second moments vary over time. Consequently, investors' portfolio can be re-optimized each period as new information becomes available, allowing for dynamic portfolio management. The main novelty we introduce lies in the use of regime switching GARCH parameters. Different regimes will produce different sets of optimal weights, according to the state of the conditional first and second moments. When a deterministic conditional International CAPM is estimated, instead, the distinct states, and the resulting optimal portfolio weights, are collapsed into one, with a consequent loss of information. In particular, since the results presented in section 4.2 indicate that a non-switching model is sluggish in reacting to shocks, it will be too slow in readjusting risk exposures and optimal portfolio weights.

State-dependent optimal international portfolio weights are obtained by, among others, Ramchand and Susmel (1998a) and Ang and Bekaert (1999). Our approach differs from that of Ramchand and Susmel in the theoretical model employed. They model excess stock market returns as an AR(1) process with switching ARCH error terms. Since such a model lacks microfoundation, the optimal portfolio weights cannot be endogenously determined. Therefore the optimal weights are not related to the theoretical model used to estimate the stochastic conditional variances and expected returns, which are input to the weights. Ang and Bekaert, on the other hand,

Panel 6B: Regression of estimated residuals on instruments

	USA	Japan	Europe	Yen	ECU	World
$\overline{R}_{global}^2$	0.003	0.006	-0.001	-0.002	0.011	0.001
$\overline{R}_{local+global}^2$		0.006	-0.002	-0.000	0.007	
$\chi_{(3),global}^2$	1.909 (0.592)	3.471 (0.325)	1.559 (0.669)	1.238 (0.744)	8.263 (0.041)	1.428 (0.699)
$\chi_{(3),local+global}^2$		3.517 (0.319)	1.066 (0.785)	2.200 (0.532)	6.795 (0.079)	
$\chi_{(4),global}^2$	2.949 (0.566)	7.792 (0.100)	1.562 (0.816)	1.626 (0.804)	8.404 (0.078)	1.800 (0.772)
$\chi_{(4),local+global}^2$		7.808 (0.099)	1.078 (0.898)	2.953 (0.566)	6.798 (0.147)	

$\overline{R}_{global}^2$  is the adjusted  $R^2$  for the regression of the estimated residuals on the three global instruments.  $\overline{R}_{local+global}^2$  is the adjusted  $R^2$  for the regression of the estimated residuals on two global instruments and the relevant local information variable. The globals are: i) The change in the 1-month Eurodollar deposit rate; ii) The change in the US default spread, defined as Moody's Baa minus Aaa long-term bond rate; iii) The change in the US term structure spread, measured by the yield to maturity on the 10 year Treasury bond in excess of the 3-month T-bill rate. The locals are the change in the real 1-month EuroYen (EuroECU) deposit rate in excess of the real 1-month Eurodollar rate, where real interest rates are computed by the Fisher formula using local inflation rates.

$\chi_{(3),global}^2$  is the Wald test statistics to test the null hypothesis that the slope regression coefficients are jointly equal to zero, in the regression containing only global instruments.  $\chi_{(3),local+global}^2$  is the corresponding test statistics when one of the globals is replaced by a local information variable.  $\chi_{(4),global}^2$  and  $\chi_{(4),local+global}^2$  are the Wald test statistics to test the null that all coefficients, including the constant term, are jointly equal to zero. Probability values are shown in parentheses.

compute optimal weights from a CRRA utility function. Since the weights are function of regime switching expected stock returns, they depend on a latent state variable. However, they do not use a market equilibrium model, hence their investor is not necessarily a representative agent.

Finally, a regime switching International CAPM will provide state dependent, and thus better, measures of conditional expected gains from cross-border diversification, optimal hedge ratios, and conditional beta and correlation coefficients.

## 6 Summary of Results and Conclusions

We have estimated a conditional version of Adler and Dumas' (1983) International CAPM with regime switching GARCH parameters, for three high-cap and highly integrated markets (USA, Europe, and Japan), at weekly frequency for the period 1986 to 1998. This approach allows for a conditional and state dependent measurement of market and currency risk premia. Prices of risk are assumed to be constant to make estimation tractable. As benchmark, the International CAPM is tested with a traditional, non-switching multivariate GARCH(1,1)-M methodology. For this model, we find that the market risk is significantly priced, whereas currency risks are not. All major market turmoils are captured, as reflected by increases of risk premia. When GARCH parameters are allowed to switch according to Hamilton's filter (1988, 1989, 1990, 1994), the price of market risk is no longer significant. However, and more importantly, stock market risk exposures are seen to react faster to shocks in returns. This may suggest that the non-switching model suffers from spuriously high persistence, with consequently sluggish reaction to shocks. In particular, when a financial crisis occurs, the conditional risk exposures seem to be underestimated, while in the aftermath they appear to be overestimated. The introduction of a regime switching model should hence improve forecasting performance, a result which is in line with previous research on switching (G)ARCH processes (see, for instance, Hamilton and Susmel 1994; Gray 1996; and Dueker 1997). We also find that in periods of financial turmoil, weight is shifting away from the GARCH towards the ARCH terms of the conditional (co)variance generating process. During such events investors, when forming their (co)variance expectations, seem to emphasize more current shocks, at the expense of the current second moments.

Our work suggests a number of future research avenues. A natural extension would be to consider time varying prices of risk, in the spirit of the recent literature on conditional CAPM (see section 2.1). This could either be done by modelling prices through a set of information variables, or

by introducing Markovian switching in the prices, like in Cappiello (2000a, 2000b). The latter approach is novel and intuitively appealing, as international investors are likely to exhibit sudden jumps in risk aversion in reaction to news. Such jumps are not always captured by the standard information set typically used in the literature to model investors' prices of risk.

Another issue to address is the optimal number of latent variables governing the switching GARCH parameters. In our framework only one latent variable is used, which facilitates estimation and also reintroduces spillovers in volatility, lost when adopting a diagonal GARCH. A more flexible model, however, should consider distinct state variables, possibly one for each market, and test if there are lead or lag effects among them. For example, a parsimonious representation could link the European and US stock markets through the state relationship  $s_t^{Europe} = s_{t-1}^{USA}$ , which assumes that the US stock market leads the European. Any systematic relationships among state variables, whether by construction or not, can be used to investigate international crisis contagion in currency and stock markets.

Furthermore, Markov transition probabilities can be made time varying through information variables, in line with recent research (see section 2.3). Switching probabilities would then become a function of investors' information set, which is intuitively appealing.

Finally, as argued in section 2.1, a conditional (International) CAPM should ideally be intertemporal to account for shifts in the investment opportunity set and the resulting hedging demands. The problem is, of course, that the number and the identities of the state variables are not given by the theory, and, moreover, that their inclusion increases the number of equations in the multivariate model.

## Appendix A1: Alternative Specifications for the Switching Model

As reported in Section 4.2, one of the GARCH parameters ( $b_2$ ) hits the upper limit of 0.9999, imposed as a necessary condition for stationarity. When the bound is released, however, the maximization algorithm does not converge. In an attempt to solve this problem, we have tried several alternative specifications for the switching model, as listed below.

1. Since  $b_2$  is associated to the Japanese stock market, we do not let the parameters linked to that asset switch. The rationale behind this attempt relies on the fact that this stock market shows a very different behavior, relative to the US and European markets, making the assumption of synchronous switching less plausible. This model converges, and  $b_2$  no longer hits the limit, but one of the Markov switching probabilities reaches the upper bound (0.9999), indicating that there is no room for two distinct regimes.
2. In line with the first attempt, which singles out the Japanese stock market for special treatment, we introduce a constant term in the mean equation for this asset. Although this representation converges,  $b_2$  still hits the upper bound.
3. Instead of individual switching coefficients  $ga_i$  and  $gb_i$  ( $i = 1, 2, 3, 6$ ), we use common switching coefficients. The vectors  $\mathbf{ga}$  and  $\mathbf{gb}$  then simplify to  $[ga\ ga\ ga\ 1\ 1\ ga]'$  and  $[gb\ gb\ gb\ 1\ 1\ gb]'$ , respectively. Estimating this model, two more parameters hit the upper limit ( $b_1$  and  $b_3$ ).
4. We allow also the Eurocurrency deposits to switch individually, so that  $\mathbf{ga}$  and  $\mathbf{gb}$  become  $[ga_1\ ga_2\ ga_3\ ga_4\ ga_5\ ga_6]'$  and  $[gb_1\ gb_2\ gb_3\ gb_4\ gb_5\ gb_6]'$ , respectively. This specification fails to converge.
5. We estimate both a GARCH(1,2)-M and a GARCH(2,1)-M specification. The former converges, but three elements of vector  $\mathbf{b}$  reach the upper limit. The latter model, on the other hand, fails to converge.
6. We let only the vector  $\mathbf{b}$  switch, i.e. individual switching coefficients  $gb_i$  multiply  $b_i$  ( $i = 1, 2, 3, 6$ ). Next, an equivalent specification for vector  $\mathbf{a}$  is estimated. In both cases one of the Markov transition probabilities approaches zero, implying that only one regime is identified. The result is not surprising, because when both  $\mathbf{a}$  and  $\mathbf{b}$  are allowed to switch, we find that they move in opposite directions (see section 4.2). Hence, if we let only one vector switch, the other cannot counter-balance the movement, effectively yielding only one possible state.

7. Instead of adopting Bekaert and Harvey's (1995) averaging technique to calculate  $\widehat{\mathbf{H}}_t$ , we use an alternative Bayesian methodology proposed by Dueker (1997). The estimation does not converge.

8. Finally, we employ a switching GARCH representation which is a multivariate extension of the univariate switching ARCH suggested by Hamilton and Susmel (1994). Such a model implies that the entire conditional covariance matrix is scaled by a matrix of switching coefficients  $\mathbf{G}_{s_t}$ , while the error terms are weighted by the inverse lagged matrix  $\mathbf{G}_{s_{t-1}}^{-1}$ . The resulting GARCH process becomes<sup>18</sup>:

$$\widehat{\mathbf{H}}_{t+1} = \mathbf{G}_{s_t} \left( \mathbf{C}\mathbf{C}' + \mathbf{A}' \mathbf{G}_{s_{t-1}}^{-1} \widehat{\boldsymbol{\varepsilon}}_t \widehat{\boldsymbol{\varepsilon}}_t' \mathbf{G}_{s_{t-1}}^{-1} \mathbf{A} + \mathbf{B}' \widehat{\mathbf{H}}_t \mathbf{B} \right) \mathbf{G}_{s_t}, \quad (\text{A1.1})$$

where all the remaining symbols are defined in the main text. As before, the money markets are assumed not to switch, hence the associated elements in  $\mathbf{G}_{s_t}$  are set equal to one. Moreover, in the second regime the switching coefficients are again normalized to one. When  $\mathbf{A}$  and  $\mathbf{B}$  are assumed to be diagonal, equation (A1.1) simplifies to:

$$\widehat{\mathbf{H}}_{t+1} = \mathbf{G}_{s_t} \left( \mathbf{C}\mathbf{C}' + \boldsymbol{\alpha}_{s_{t-1}} \boldsymbol{\alpha}_{s_{t-1}}' \odot \widehat{\boldsymbol{\varepsilon}}_t \widehat{\boldsymbol{\varepsilon}}_t' + \mathbf{b}\mathbf{b}' \odot \widehat{\mathbf{H}}_t \right) \mathbf{G}_{s_t}, \quad (\text{A1.2})$$

where  $\boldsymbol{\alpha}_{s_{t-1}}$  is defined as  $\mathbf{a}(\div) \mathbf{g}_{s_{t-1}}$  and  $(\div)$  denotes the Hadamard matrix division. As usual vector  $\mathbf{a}$  and  $\mathbf{b}$  contain the diagonal elements of the matrices  $\mathbf{A}$  and  $\mathbf{B}$ , respectively, while  $\mathbf{g}_{s_{t-1}}$  is the vector of the diagonal elements of the matrix  $\mathbf{G}_{s_{t-1}}$ . Notice that there are four possible states of  $\widehat{\mathbf{H}}_{t+1}$ , due to the fact that the coefficients of the matrix  $\mathbf{G}_{s_t}$  switch between two regimes at time  $t$ , whereas those of the vector  $\boldsymbol{\alpha}_{s_{t-1}}$  do so at  $t-1$ . First, the process (A1.2) is estimated assuming common switching factors for the stock markets, i.e.  $G_{ij,s_t} = G_{s_t}$  for  $i, j = 1, 2, 3, 6$ . This model does not converge. Next, we allow for individual switches of the stock markets, i.e.  $G_{ij,s_t}$  for  $i, j = 1, 2, 3, 6$ . In this case one of the Markov switching probabilities reaches zero, an indication that two distinct states do not exist.

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<sup>18</sup>The derivation of the model is omitted here, but is available from the authors upon request.



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