Vulnerable Options, Risky Corporate Bond and Credit Spread*

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Abstract

In the current literature, the focus of credit risk analysis has been either on the valuation of risky corporate bond and credit spread, or on the valuation of vulnerable options, but never both in the same context. There are two main concerns with existing studies. First, corporate bonds and credit spreads are generally analyzed in a context where corporate debt is the only liability of the firm and firm's value follows a continuous stochastic process. This set-up implies a zero short term spread, which is strongly rejected by empirical observations. The failure of generating non-zero short term credit spreads may be attributed to the simplified assumption on corporate liabilities. Since a corporation generally has more than one type of liabilities, modelling multiple liabilities may help to incorporate discontinuity in firm's value and hence lead to realistic credit term structures. Second, vulnerable options are generally valued under the assumption that a firm can fully payoff the option if the firm's value is above the default barrier at option's maturity. Such an assumption is not realistic since a corporation can find itself in a solvent position at option's maturity but with assets insufficient to payoff the option. The main contribution of this study is to address the above concerns. The proposed framework extends the existing equity-bond capital structure to an equity-bond-derivative setting, and encompasses many existing models as special cases. The firm under study has two types of liabilities: a corporate bond and a short position in a call option. The risky corporate bond, credit spreads and vulnerable options are analyzed, and are compared with their counterparts from previous models. Numerical results show that adding a derivative type of liability can lead to positive short term credit spreads and various shapes of credit spread term structures, which are not possible in previous models. In addition, as a surprising result we find that vulnerable options need not always be worth less than their default free counterparts.

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1. Introduction

The focus of credit risk analysis has been either on the valuation of risky corporate bonds and credit spreads, or on the valuation of vulnerable options, but never on both in the same context. Risky corporate bonds and credit spreads have been modelled by Black and Scholes (1973) and Merton (1974), and extended by Black and Cox (1976), Longstaff and Schwartz (1995), Briys and de Varenne (1997), and others. Studies of vulnerable options are pioneered by Johnson and Stulz (1987), and subsequently advanced by Hull and White (1995), and Jarrow and Turnbull (1995). The existing studies can be improved in several aspects. First, corporate bonds and credit spreads are generally analyzed in a context where corporate debt is the only liability of the firm and firm's value follows a continuous stochastic process. The obvious implication of this set-up is a zero short term credit spread, which is strongly rejected by empirical observations (see Fons 1994, Jones, Mason and Rosenfeld 1984, Sarig and Warga 1989). Since a corporation generally has more than one type of liabilities and one maturing liability may cause a sharp decrease in firm's value, modelling multiple liabilities may help to incorporate discontinuity in firm's value and hence lead to non-zero short term credit spreads.

Second, vulnerable options are priced under either the assumption that the option is the only liability of the firm (e.g., Johnson and Stulz (1987)), or the assumption that the option is fully paid off if the firm's value is above the default barrier at option's maturity (e.g. Hull and White (1995)). Both assumptions are questionable. On the one hand, most firms have debts in their capital structure and contingent liabilities (in the form of derivatives) are only part of the total liability. On the other hand, it is not appropriate to evaluate an option's vulnerability by focusing only on technical solvency since a corporation can find itself in a solvent position at option's maturity but with assets insufficient to payoff the option. To illustrate, suppose the value of the firm's assets until option's maturity has always been above the default barrier which is \$40. Further suppose that the firm's value is \$50 at option's maturity. If the option is \$20 in the money, the firm is "threshold-solvent" but unable to pay \$20 in full to the option holder. The downfall of Barings Bank serves as a convincing illustration — the default on debentures was purely due to the large loss on derivatives positions.

Third, risky corporate bonds (or corporate credit spreads) are usually analyzed separately from other liabilities, especially short positions on derivatives. However, many firms take derivative positions and at the same time have outstanding corporate debts. It will be useful to examine how risky corporate bonds affect the valuation of vulnerable derivatives, and vice versa. As a matter fact, some authors (e.g., Bodnar, Hayt, and Marston (1998), Howton and Perfect (1998), and Levich, Hyat and Ripston (1998)) have demonstrated the increasing usage of derivatives by financial and non-financial firms. In a wide survey of non-financial firms, Bodnar, Hayt and Marston (1998) found that 50% of the responding firms report use of derivatives. "Of the derivative users, 42% indicated that useage has inreased over the previous year" (Bodnar, Hayt and Marston (1998), p71). Moreover, the implementation of FAS133 requires that firms report their derivative positions at fair value on their balance sheet, which effectively crystalizes the contribution of any short positions on derivatives to a firm's overall capital structure. In additions, as pointed out by a reviewer, firms also take indirect derivative postions. For example, an investment bank may hold an option position in a takeover target; a financing firm frequently issues letter of credit which represents contingent liabilities, a large conglomerate may act as a third party loan guarantor which again results in contingent liabilities, and so on.

The main objective of this study is to overcome the aforementioned shortcomings in the existing literature. We propose a framework which incorporates two types of liabilities: a corporate bond which represents the conventional debt of the firm and a short position in a call option which represents contingent liabilities. The default barrier is stochastic with respect to the interest rate and can be a function of either the initial option's value or the market value of the option. Under each default barrier specification, we also examine two alternative settlement rules. When the default barrier is determined by the market value of the option, the option buyer essentially imposes a mark-to-market style of covenant. The framework is quite general and encompasses many existing models as special cases, including Merton (1974), Black and Cox (1976), Johnson and Stulz (1987), Longstaff and Schwartz (1995), and Briys and de Varenne (1997). Numerical analyses demonstrate that including an additional liability can generate positive short term credit spreads. Under reasonable parameter values, credit spreads can exhibit upward-, downward- and hump-shaped term structures. In contrast, within the existing models, a downward credit spread term structure is possible only when the firm is already bankrupt. It is shown that bond maturity, moneyness of the option (or significance of the other type of liability), default barrier requirement, correlation between firm's value and the optioned stock price all play a role in determining the level and the shape of credit spread curves. Moreover, we show that a vulnerable European option may be worth more than its default-free counterpart if the option holder is paid off according to a particular claim rule upon default.

The paper is organized as follows. Section 2 outlines the model settings and discusses possible payoff rules under different scenarios for call holders, bondholders and shareholders. Section 3 presents simulation results for vulnerable options, risky bonds and credit spreads under reasonable parameters, and compares our results with those generated by some of the existing models. Section 4 concludes the paper. Tables and graphs are collected in appendices.

2. The Model

2.1. Model Set-up

Consider a firm which has, as liabilities, a zero-coupon bond with maturity T_b and face value F and a short position on a call option written on another firm's stock with strike price K and maturity T_c ($< T_b$). [1] We assume a continuous-time economy where financial markets are complete and frictionless so that we can apply Harrison and Kreps' (1979) Equivalent Martingale Pricing Principle to price the securities under consideration.

The short-term riskless interest rate r_t at time t is assumed to evolve according to a Vasicek's (1977) type of mean-reverting process under the equivalent probability measure:

$$\mathrm{d}r_t = a(b - r_t)\mathrm{d}t + \sigma_r \mathrm{d}z_{1t}, \tag{2.1}$$

where a, b and σ_r are constant and z_{1t} is a standard Wiener process. Without loss of generality, we assume a zero market price of risk. Under the above short-rate process, the default-free zero

[1] We use a call option as an example. The analysis can also be performed for other types of derivatives.

coupon bond $P(t,T)$ maturing at time T is governed by the following process

$$\frac{\mathrm{d}P(t,T)}{P(t,T)} = r_t \mathrm{d}t - \sigma_r \frac{1-e^{-a(T-t)}}{a} \mathrm{d}z_{1t}. \tag{2.2}$$

The firm's asset value under the equivalent probability measure is assumed to follow

$$\frac{\mathrm{d}A_t}{A_t} = r_t \mathrm{d}t + \sigma_A \mathrm{d}z_{2t}, \tag{2.3}$$

where σ_A is the standard deviation of the assets return and z_{2t} is a Wiener process. The correlation between the short-term riskfree rate and asset value is ρ_{Ar}. The firm value is assumed to be independent of the capital structure of the firm. Finally, the price of the optioned stock under the equivalent probability measure is described by the following stochastic process:

$$\frac{\mathrm{d}S_t}{S_t} = r_t \mathrm{d}t + \sigma_s \mathrm{d}z_{3t}, \tag{2.4}$$

where σ_s is the instantaneous volatility and z_{3t} is a standard Wiener process. The correlation between the short-term riskfree rate and the optioned stock price is ρ_{sr} while that between the optioned stock price and the firm value is ρ_{sA}.

The default barrier or threshold level at time $t, v(t)$, is specified as

$$v(t) = \begin{cases} \alpha_1 FP(t,T_b) + \alpha_2 C & \forall \quad t < T_c, \\ \alpha_1 FP(t,T_b) & \forall \quad T_c \le t < T_b, \\ F & for \quad t = T_b. \end{cases}$$

As soon as the firm value A_t falls below $v(t)$, the safety covenant would trigger bankruptcy. Prior to the option's maturity, this threshold represents the sum of the minimum requirements imposed by bondholders and option holders. After the option's expiration, the threshold represents bondholders' minimum requirement. Depending on the specific features of the covenant, C could be the initial value, C_0 or the (varying) market value C_{mkt} of an otherwise default-free option. In the former case, the option holders impose a fixed amount of default protection while in the latter, the protection amount varies according to the market value of the option, which amounts to mark-to-market margin requirements. The parameters α_1 and α_2 represent the degree of protection for the two classes of liability holders, and are assumed to be positive constants. The maximum value of

α_1 is 1.0, corresponding to full protection to bond holders. However, α_2 can be bigger than 1.0, especially when the initial value of the option, C_0 is used in the covenant specification. Note that the default barrier is stochastic, with the bond portion depending on the evolution of the short-term riskfree interest rate, and the option portion depending on both the interest rate and the optioned stock's price when the covenant is based on C_{mkt}.

Next, we specify two distribution rules upon default. The first approach is the proportional distribution rule based on the default barrier requirements while the second based on the market values of the corresponding default-free instruments. Under the default barrier distribution rule, the payoff proportion to the option holder is based on the initial value of the option, except when the default occurs at the option maturity in which case we use the option's intrinsic value $\max(S(t_2)-K, 0)$ to calculate the proportions. To allow for potential violations of the strict priority rule at default resolutions, we adjust the payoff proportions for the bond holders and option holders respectively by fractional numbers, γ_1 and γ_2. When $\gamma_1 = 1$ and $\gamma_2 = 1$, there is no leakage and the two classes of creditors receive the full amount they are entitled to. Denote $T_{A,v}$ as the first passage time for the process A to go through the barrier v, then $T_{A,v} = \inf\{t \geq 0, A_t = v(t)\}$ $\forall$ $0 < t \leqslant T_b$ and $T_c < T_b$. The payoffs are summarized in Table 1.

The payoffs under Scenarios 1, 3, and 4 are straightforward and easy to understand. However, under Scenario 2 when $A(T_c) \leq v(T_c)+[S(T_c)-K]^+$, although the asset value is above the threshold, upon the exercise of the call, the remaining value of the asset is below the required threshold by the bondholder. This is a case where the firm meets the covenant requirements before option's maturity but is triggered into default by the exercise of the option. Barings' bankruptcy serves as the best example to illustrate the point. Barings was financially strong and solvent before the huge loss on derivatives trading was revealed. The unfortunate bankruptcy was triggered by the large obligations resulted from the Nikkei index futures. Our paper will represent the initial effort in addressing the consequences of such a possible outcome on the credit spreads and vulnerable option values. We will show in the numerical analysis that incorporating this possible outcome helps us to generate positive short term credit spreads and a downward sloping term structure of credit spreads, which the existing models can generate only if the firm is already insolvent.

Table 1: Payoff Descriptions for Difference Scenarios

Scenario 1: Default Prior to Option's Maturity: $T_{A,v} < T_c$

	Default Barrier Rule	Market Value Rule
Call Holder	$\alpha_2 C\gamma_2$	$w_1\gamma_2 A(T_{A,v})$
Bondholder	$\alpha_1 FP(T_{A,v}, T_b)\gamma_1$	$(1-w_1)\gamma_1 A(T_{A,v})$
Shareholder	0	0

Scenario 2: No Default Prior to Option's Maturity

	$A(T_c) \le v(T_c)$ (**default**)		$A(T_c) > v(T_c)$		
			$A(T_c) \le v(T_c) + [S(T_c) - K]^+$ (**default**)		$A(T_c) > v(T_c) + [S(T_c) - K]^+$
	Default Barrier Rule	Market Value Rule	Default Barrier Rule	Market Value Rule	
Callholder	$w_2\gamma_2 A(T_c)$	$w_3\gamma_2 A(T_c)$	$w_2\gamma_2 A(T_c)$	$w_3\gamma_2 A(T_c)$	$[S(T_c) - K]^+$
Bondholder	$(1-w_2)\gamma_1 A(T_c)$	$(1-w_3)\gamma_1 A(T_c)$	$(1-w_2)\gamma_1 A(T_c)$	$(1-w_3)\gamma_1 A(T_c)$	
Shareholder	0	0	0	0	

Scenario 3: Default Prior to Bond Maturity: $T_c < T_{A,v} < T_b$

Bondholder	$A(T_{A,v})\gamma_1$
Shareholder	0

Scenario 4: No Default Prior to Bond Maturity

	$A(T_b) \le F$	$A(T_b) > F$
Bondholder	$A(T_b)\gamma_1$	F
Shareholder	0	$A(T_b) - F$

Note: 1. $\max[S(T_c) - K, 0] = [S(T_c) - K]^+$

2. $w_1 = \frac{C_{mkt}}{FP(T_{A,v}, T_b) + C_{mkt}}$, $w_2 = \frac{\alpha_2 C}{\alpha_1 FP(Tc, T_b) + \alpha_2 C}$, $w_3 = \frac{[S(T_c) - K]^+}{FP(T_c, T_b) + [S(T_c) - K]^+}$.

3. When the default barrier is based on the initial option value, $C = C_0$; when the default barrier is based on the market value of the option, $C = C_{mkt}$.

2.2. Discussions

We have specified the option's maturity to be shorter than the bond's, which by and large reflects reality. [2] One could easily study a case where the opposite applies. The only conceivable practical scenario corresponding to this setup would be a case where a large amount of corporate debt is maturing. In such a case, debt default is a less important issue given the imminent maturity, but there could be implications for vulnerable options. Such a case is straightforward to study, but is omitted for brevity.

An acute reader may realize that we do not explicitly specify a recovery rate as many other authors have done. In our model, prior to the option's maturity, the recovery rate is stochastic and endogenous, and dependent on the interest rate and the severeness of the bankruptcy. This is an important improvement over the existing literature which mostly employs a constant exogenous recovery rate. (As apparent in Table 1, the recovery rate beyond the option's maturity will be $\alpha_1\gamma_1$ before the debt's maturity, and between $\alpha_1\gamma_1$ and 1.0 at the debt's maturity.)

Several additional properties of our setup need to be pointed out. First, when the covenant is based on the market value of the option, C_{mkt}, the two distribution rules become identical if $\alpha_1 = \alpha_2$. In the general case where $\alpha_1 < \alpha_2$, it can be seen that the market-value-based distribution rule would be favorable to bond holders and detrimental to option holders, when compared with the barrier-based distribution rule. Second, as long as $\alpha_1 \neq 1$ the bond will always be risky, regardless of the level of other parameters. This mimics the reality well. Third, under the default-barrier-based distribution rule and the market value (of option) based covenant, the option would be completely default-free if $\gamma_2 = 1$ and $\alpha_2 = 1$. This is as if the option is marked to market continuously and margin deposits are adjusted accordingly. With OTC derivatives, this is highly unlikely, although counterparties have lately moved towards a fuller protection in the context of credit risk management. Our setup is capable of accommodating this reality. For example, under the market-value-based distribution rule, even if $\gamma_2 = 1$ and $\alpha_2 = 1$, the call option is still vulnerable. This situation is equivalent to the following: daily re-valuation of the option is indeed

[2] Survey results from Bodnar, Hayt and Marston (1998) indicate that most firms use derivatives with short maturities. For example, 77% of the foreign currency derivatives being used have an original maturity of 91 to 180 days.

performed and covenant requirement re-adjusted, but margin deposits are either not required or not adjusted as frequently as the re-valuation. This is a realistic scenario in light of the wide-spread implementation of VaR (which means frequent re-evaluation and re-assessment of counterparty risk), and the non-existence of an established margin mechanism for OTC products.

Thanks to its general and realistic features, our model can be reduced to many previous models as special cases. For vulnerable options, setting the default barrier to zero (by setting $F = 0$ and $\alpha_2 = 0$) and the priority parameter to one ($\gamma_2 = 1$) leads to the model by Johnson and Stulz (1987). For risky debts, our model encompasses several existing models. First, our setup reduces to the simple framework of Merton (1974) by setting the option position to zero ($S = 0$) and removing the bond covenant protection (by setting $\alpha_1 = 0$). [3] Second, the model of Black and Cox (1976) assumes a constant riskfree interest rate and no violation of the strict priority rule, which can be achieved in our model by setting the option position to zero ($S = 0$), and $a = 0$, $\sigma_r = 0$, and $\gamma_1 = 1$. Third, Longstaff and Schwartz (1995), using the same stochastic processes as in (2.1) and (2.3), derived a closed form formula for the risky bond with a constant default barrier. In our framework, this is equivalent to setting the option position to zero ($S = 0$) and the default barrier to a constant, $v(t) = K_0$ ($\forall\ t \leqslant T_b$). Finally, Briys and de Varenne (1997) extended the model by Longstaff and Schwartz (1995) by allowing the default barrier to vary according to the market value of an otherwise riskfree bond. In spirit, our model is close to Briys and de Varenne's, although the latter studies risky bonds only. We can obtain their model by setting $S = 0$. It is worth repeating that none of the aforementioned models can generate non-zero short term spreads and a downward sloping credit term structure for a solvent firm, although such features have been observed empirically (Fons (1994), and Sarig and Warga (1989)).

Once we specify the stochastic processes and the payoff rules, the value of the vulnerable option and the defaultable bond can be expressed by way of risk-neutral discounting. Unfortunately, the valuation expressions do not admit closed-form solutions due to the complex recovery rate specifications or the distribution rules. As a result, we must resort to Monte Carlo simulations which are delineated in the following section.

[3] Throughout the discussions here, "seeting the option position to zero" means $S = 0$, which is different from $\alpha_2 = 0$. In the latter case, although the option's position does not affect the default barrier, paying off the option at maturity will alter the asset value process, and hence the bond default dynamics.

3. Numerical Analysis

The stochastic processes for the firm value, riskfree interest rate and the stock price are discretized into daily interval by assuming 250 days in a year. Correlated paths of the three variables and that of the default barrier are then generated. At the beginning of each daily interval, the firm value is compared against the default barrier. If default occurs, then the two classes of liability holders are paid off according to the pre-specified distribution rule, and a new run is started, and so on. Each numerical estimate is based on 10,000 runs. To reduce the simulation variance, we employ both the antithetic method and the control variate technique.

To apply the antithetic method, for each regular path of the variable in question, we generate a companion path which uses the same random innovations as used by the regular path, but reverse the signs. For each run, the simulated value in question, is simply the average of the two simultaneously generated values. [4] To apply the control variate technique, we need to choose a "control variate" for the risky bond and the vulnerable option. For the risky bond, the control variate is the defaultable bond studied by Briys and de Varenne (1997) which admits a closed-form formula; for the vulnerable option, the control variate is the default-free counterpart whose formula was derived by Merton (1973). The difference between the theoretical value and the simulated value of the control variate will then be used to adjust the simulated value of the instrument in question. For details on the two variance reduction techniques, please consult Hull (2000).

The parameter values for the simulation are chosen such that they are consistent with empirical observations. Without loss of generality, we scale the current value of the firm to $A_0 = 100$. The instantaneous volatility of the asset value or firm value is set at $\sigma_A = 0.2$, consistent with empirical findings of Jones, Mason and Rosenfeld (1984) and the discussions in Leland and Toft (1996). For the interest rate process, the mean-reversion speed and the instantaneous volatility are set according to estimation results of Chan, Karolyi, Longstaff and Sanders (1992), and being

[4] Unlike a typical simulation where the antithetic path always mirrors the regular path, in our setting, the two paths may terminate at different times since we are essentially simulating the first passage time. Nonetheless, the "mirroring effect" or variance reduction is achieved at least for the time period when both paths are in the solvent region.

consistent with the values used by Briys and de Varenne (1997). Specifically, $a = 0.2$ and $\sigma_r = 0.02$. The spot interest rate and the long-run reversion target are both set at 10% ($r_0 = b = 10\%$) to produce roughly a flat term structure of the riskfree rate, which will simplify interpretations of the risky term structure. The instantaneous volatility of the stock return is $\sigma_S = 0.25$, the level for a typical stock. In accordance with Footnote 3, the option's maturity T_c and the bond maturity T_b are set at 0.5 years and 5 years respectively for the base case, and other values are examined in comparative analysis. The current stock price S_0 is set at 100 so that when the option is at the money, it is about 10% of the firm's asset value. Realizing that equity and asset values tend to be negatively correlated with interest rates, we set the three correlations at $\rho_{sA} = 0.3$, $\rho_{sr} = -0.4$, and $\rho_{Ar} = -0.25$, consistent with Briys and de Varenne (1997), The base case values for the covenant protection parameters are $\alpha_1 = 0.85$ and $\alpha_2 = 1.00$, and the priority rule parameters are $\gamma_1 = 0.90$ and $\gamma_2 = 1.00$. Finally, the face value of the bond is set such that the debt ratio is 50% initially: $F = 0.5A_0/P(0, T_b)$.

To enhance our insights, we will compare our results with those of the existing models. For risky bonds and credit spreads, since our model is a direct extension of Briys and de Varenne (1997) who in turn extended the previous models, we will include the credit spreads from their model for comparison. The credit spread generated in our model will always be larger than that in Briys and de Varenne (1997), since we introduce another liability, the option. For vulnerable options, since our framework is a direction extension of Johnson and Stulz (1987), we will use their model as a comparator. Notice that in the framework of Johnson and Stulz (1987), the vulnerable option only depends on the firm value dynamic and is assumed to be unaffected by the capital structure of the firm. Therefore our model extends that of Johnson and Stulz (1987) in three ways. First, we allow default prior to the option's maturity; second, we allow bonds to compete with the option for payoffs at the option's maturity; and third, we allow a stochastic interest rate. Since the last extension is a straightforward generalization of the constant interest rate assumption, we will assume that it is already a feature of Johnson and Stulz (1987). In other words, when we refer to Johnson and Stulz (1987), we will mean their model implemented with a stochastic interest rate.

We will examine various combinations of modelling parameters in order to fully understand the behavior of vulnerable options and credit spreads. Schematically, we first start with two main dimensions: whether the option component of the default barrier ($\alpha_2 C$) is based on the initial option value C_0 or the market value, C_{mkt}. With each barrier specification, two distribution rules at default are possible: either default barrier based or market value based, as outlined in Table 1. Under each combination we examine how vulnerable options and credit spreads are affected by such variables as bond maturity, capital structure, extent of covenant protection, and various correlations. In case of similar results we report only the representative for brevity. Considering that daily mark-to-market and margin requirements are not yet an established practice for OTC derivatives, we start first with the barrier specification based on the initial value of the option. Later, in Section 3.2, we study the market value based default barrier, and compare the two.

3.1. Default Barrier Based on Initial Option Value

Here the two distribution rules, default barrier rule and the market value rule, generate very similar results. For brevity, we only present the results under the default barrier rule.

3.1.1. Vulnerable Options and Term Structure of Credit Spreads

We first study how the credit spreads vary across maturities and simultaneously examine the value of the vulnerable option. To this end, in Exhibit 1, we vary the bond maturity from one year to ten years, and accordingly adjust the face value of the bond so that the debt ratio remains at 0.5. Given our assumption of firm value's invariance to the capital structure, the sum of the defaultable bond value, the vulnerable option value, and the equity value is always $A_0 = 100$ if there is no "leakage". Whenever there is a leakage (i.e. $\gamma_1 \neq 1$), we will credit the amount of the bond payoff leakage to equity value so that the conservation of value is obtained. This is done mainly as a way of double checking the numerical accuracy.

The effects of debt maturity are shown in Panel A and Panel B where the option's maturity is fixed at 0.5 years. There are several interesting observations. First, the default-free call and the vulnerable call based on Johnson and Stulz (1987) are both independent of the bond maturities, as they should be. Second, in both panels, it is seen that the value of the default-free call is higher than that of the vulnerable call based on Johnson and Stulz (1987), which in turn is higher than the vulnerable call's value generated from our model. The option is the most vulnerable in our model since default can occur not only at option's maturity, but also at anytime before the option's maturity. Although the results make intuitive sense, as we will see later, the vulnerable option in our model can be worth more than its counterpart in Johnson and Stulz (1987), or even its default-free counterpart. Nonetheless, both panels show that the vulnerable option's value is not very sensitive to the bond maturity. Third, the credit spread from our model is higher than its counterpart from Briys and de Varenne (1997), as expected. When the option is in-the-money, the credit spread is much larger, which makes intuitive sense. In this case, the model by Briys and de Varenne (1997) will significantly under-estimate the credit spreads. The importance of incorporating non-debt liabilities becomes obvious. Fourth, the shape of the credit spread term structure depends on the moneyness of the option. It is upward sloping when the option is at-the-money, and downward sloping when the option is deep in-the-money.[5] As discussed earlier, in previous models, including that of Briys and de Varenne (1997), downward sloping term structure of credit spreads is possible only when the firm is already insolvent. In our model, a short position on an option can easily lead to the empirically observed result.

The above finding begs a natural question: what if the option's maturity increases with the bond's? To answer this question, we re-run the simulations for two cases: to maintain the option's maturity at 1/4 of the bond's in one case and at 1/2 of the bond's in the other. As the two figures in Exhibit 1 show, sizable short term credit spreads are still present when the option is in-the-money. For example, when the option's maturity is maintained at 1/4 of the bond's, the one-year credit spread is around 5% when the stock price is $130. In addition, the option's moneyness also determines the varieties of the term structure's shape. Downward sloping, upward sloping and humped structures are all observed. In contrast, the term structure generated from Briys and de Varenne (1997) is always upward sloping. Again, given the empirical observations of flat or downward sloping credit spread term structures by Saig and Warga (1989) and Fons (1994), our model has proved its versatility and potential.

[5]Notice that when we perform simulations for in-the-money cases, we set the initial barrier using the value of an at-the-money option. This also holds for all subsequent simulations involving different moneyness situations. Essentially we are simulating situations where the barrier was set sometime ago and the option's moneyness has since changed. In addition, we omit the results for out-of-the-money cases since our model generates virtually the same results as that of Briys and de Varenne (1997), due to the minimal effect of the option's position.

3.1.2. Effects of Capital Structure

In our model, capital structure or debt ratio is defined as the ratio of the market value of an otherwise riskfree bond over the current firm value: $F * P(0, T_b)/A_0$. To study its impact, we repeat the calculations for Panel A in Exhibit 1 for different debt ratios and report the results in Exhibit 2. The credit spread is very small when the debt ratio is 0.3. When the debt ratio is 0.7, credit spreads are sizeable in both models. Here, our model generates a downward sloping credit spread term structure, which is in contrast with the upward sloping curve apparent in Panel A of Exhibit 1 with a debt ratio of 0.5. The above implies that capital structure determines not only the level but also the shape of the credit term structure. Moreover, with a debt ratio of 0.3, the credit term structure is U-shaped in our model but upward sloping in Briys and de Varenne (1997); with a debt ratio of 0.7, the term structure is humped in Briys and de Varenne (1997). In Panel C of Exhibit 2, we vary the debt ratio between 0.1 and 0.9. It is seen that the value of the vulnerable option generally decreases and the credit spread increases, as expected. However, we notice a peculiar phenomenon at a debt ratio of 0.9 whereby the vulnerable option's value is higher than its default-free counterpart and its vulnerable counterpart in Johnson and Stulz (1987). The "price inversion" with respect to the vulnerable counterpart in Johnson and Stulz (1987) also exists when the debt ratio is 0.1, 0.2 and 0.3. We will show in Exhibit 6 that it is entirely possible for a vulnerable option in our model to be worth more than its default-free counterpart.

3.1.3. Degree of Covenant Protection

Recall that the strictness of the covenant is captured by the two parameters, α_1 and α_2. Since derivatives' position (as a percentage of the firm's asset value) is usually smaller than debt's, we still set $\alpha_2 = 1$, and only examine the effect of varying α_1. Exhibit 3 reports the results. It is seen that, in both models, the credit spread decreases as the degree of protection increases. When a full protection is in place, the credit spread is not zero in either model due to the violation of the strict priority rule (i.e. $\gamma_1 = 0.9$). In our model, even if $\alpha_2 = 1$ and $\gamma_1 = 1.0$, the bond is still risky because default can be triggered by option's position in which case bond holders' proportional claim to the asset value may be less than the market value of the riskfree bond if the option is deep in-the-money. Similar observations can be made for other moneyness situations, for which we have plotted at the bottom of the exhibit the credit spread term structures. It is interesting to notice that the credit spread is almost constant for most of the moneyness situations when $\alpha_1$is not very high. This is especially true in the framework of Briys and de Varenne (1997). Intuitively, when the default barrier is lower than a certain threshold, the possibility of default before option's maturity is almost zero (and hence α_1 ceases to play a role), and the spread is almost entirely due to defaults at option's maturity. In the case of Briys and de Varenne (1997), the constant spread is almost entirely due to the violation of the strict priority rule.

In the framework of Johnson and Stulz (1987), the vulnerable option is not affected by the bond covenant protection. However, in our model, it is seen that the vulnerable option's value goes down as the bond covenant protection increases. This is a result of a subtle trade off. When the bond covenant protection increases, the overall default barrier is raised, and as a result, both the bond holders and the option holders enjoy better protection against dramatic losses. However, since option holders' covenant protection remains unchanged, an increase in α_1 means a higher claim proportion for bond holders (and a lower proportion for option holders) at default. It is apparent from the exhibit that the latter effect dominates the former. An immediate implication is that option holders should not only ensure a higher overall default barrier, but also ensure that their claim in case of default is commensurate with the default barrier.

3.1.4. Effects of Correlation between Firm Value and the Optioned Stock Price (ρ_{sA})

Both option holders and bond holders should be concerned about how the firm value and the optioned stock price are correlated. For option holders, a knowledge of the impact of the correlation on the option value can help them structure the covenant initially; it is also true for bond holders in that they have equal claim priority as the option holders. To gain some insights, we repeat the previous simulations for two additional values of ρ_{sA}: 0.0 and -0.3, and plot the results in Exhibit 4, together with the case for $\rho_{sA} = 0.3$.

First, we notice the usual patterns whereby the credit term structure generally slopes upwards when the option is at- or near-the-money, and downward when the option is deep in-the-money. Second, as the correlation moves toward zero and become more negative, the credit spread becomes bigger for a particular maturity. Although not reported, the vulnerable option's value goes down as the correlation becomes negative. The results make intuitive sense. When the firm value and stock price are negatively correlated, a higher stock price is likely to be accompanied by a lower firm value, in which case both groups of liability holders stand to lose more than otherwise in case of default. However, a positive correlation will on average ensure that any gain on option values is supported by an increase in the firm value, which will make default less likely, and as a result, both types of liability holders benefit. The above implies that a relatively lax covenant is warranted if the firm value is positively correlated with the optioned stock price, and vice versa.

It is also apparent from the figures that the impact of the correlation diminishes as the bond maturity increases. This makes intuitive sense because the option's maturity is fixed at 0.5 years. As the bond maturity increases, the effect from option's position is being spread out. Notice that the credit spread in Briys and de Varenne (1997) is independent of ρ_{sA}.

3.1.5. Effects of Correlation between Firm Value and Interest Rate (ρ_{Ar})

Insofar as credit spread is jointly affected by the behavior of the firm value and the interest rate, it would be useful to see how the co-movement of the two variables affects the credit spreads and vulnerable options. To this end, we repeat previous simulations for various levels of ρ_{Ar}, and report the result in Exhibit 5. It is seen that the correlation ρ_{Ar} does not affect the shape of the credit term structure in any significant way, although it does slightly affect the level of credit spreads, as it affects the vulnerable option's value. Specifically, as the correlation moves from negative to positive, the vulnerable option's value in our model decreases, while the credit spread increases, or equivalently, the bond price decreases. Let us first examine the bond. With a positive correlation, when the interest rate is high, the firm value tends to be high, but the bond value is low. In this case, the higher firm value does not benefit bond holders very much because their claim is lower anyway. But when the interest rate is low, the bond value is high and firm value tends to be low, and default is more likely. If default does occur, bond holders stand to lose more because of the lower firm value. Therefore, a positive correlation between firm value and interest rate leads to situations where helps are bountiful when not required, and scarce when needed.

As for the vulnerable options in our model, it is first to be recognized that a higher interest rate leads to a higher call option value, other things being equal. However we have specified a negative correlation between the stock price and the interest rate (i.e., ρ_{sr} = -0.4), which means a higher interest rate tends to be associated with a lower stock price, and hence a lower option value. If the second effect dominates the first, then we tend to see a lower option value associated with a higher interest rate. Now, a positive correlation between firm value and interest rate would imply a higher firm value with a higher interest rate, and vice versa. Combining the above, we obtain a similar explanation for the pattern of the vulnerable option's value as that for the bond's: helps are not needed when available, and absent when desired. This is why we see a downward pattern in the option price when the correlation increases. Finally, within the model of Johnson and Stulz (1987), the vulnerable option's value goes up slightly as the correlation increases, and this is because the first effect dominates in this case, thanks to the fact that there is no interim default.

3.1.6. A Closer Examination of Vulnerable Options

Up to this point, we have been studying vulnerable options and defaultable bonds when the option is either at-the-money or in-the-money, and the option's maturity is relatively short. In those cases, the option component of the default barrier is either close to or lower than the market value of the option (most of the time). When default does occur, option holders tend to receive a settlement less than the market value of the option (under the default barrier distribution rule which we are using). However, we sometimes observe in previous exhibits that the vulnerable option is worth more than its default-free counterpart. As apparent in Table 1, option holders can potentially receive a settlement worth more than the market value of the option. This occurs when a default is triggered, yet the market value of the option is much lower than the initial value, C_0. A plausible corresponding scenario would be one where the option is struck at-the-money and default barrier is set accordingly, but the option becomes out-of-the-money subsequently. To confirm this, we calculate option values and credit spreads for an out-of-the-money option with different maturities, and report them in Exhibit 6. For comparison, we also calculate the same for an at-the-money option. (While we vary the option's maturity from 0.5 years to 5 years, the bond's maturity is kept at 5 years.) It can be seen that when the option is out-of-the-money, it is worth more than its counterpart in Johnson and Stulz (1987) for all maturities, and more than the default-free counterpart when the maturity is beyond 2.0 years. When the option is at-the-money, it is worth less than its default-free counterpart for all maturities, but still more than its counterpart in Johnson and Stulz (1987) when the maturity is beyond three years. [6]

Bond holders do not enjoy this luck in the setup, since their contribution to the barrier specification is fully market value based. That is why we did not observe a single negative credit spread. Unless covenants are fully market value based for OTC derivatives and margins are posted accordingly, which is unlikely to achieve in practice, vulnerable options can always be potentially less vulnerable!

Finally, notice that the credit spread becomes bigger as the option's maturity becomes longer. This reflects the bigger liability other than the debt incurred by the firm. This is in contrast with the constant spread of 0.42% produced by the model of Briys and de Varenne (1997).

3.2. Default Barrier Based on Market Value of Option

So far, the analyses are based on default barriers that depend on the initial value of the option. If the barrier is allowed to depend on the market value of the option, then there will be altogether four possible combinations to consider — two default barrier specifications and two distribution rules.

Some general discussions are in order. First, the bond is risky under all combinations except one: when the default barrier is based on the initial value of the option and the distribution rule is market value based. In this case, it is possible that a default is triggered when the option is deep out-of-the-money, and the settlement payoff to the bond holders is higher than the market value of the riskfree counterpart. This scenario of course doesn't make much sense. To avoid this, we arbitrarily stipulate that when the default barrier is breached, the firm is dissolved only if the firm value is lower than the default barrier based on the market value of the option at that point. Second, as far as option holders are concerned, it can be determined from Table 1 that, when $\alpha_2 = 1$ the default barrier distribution rule is always preferred to the market value distribution rule, no matter how the default barrier is specified. But with the same distribution rule, it is not entirely clear as to which default barrier specification is preferable. Third, the opposite is true for

[6] The intuitive reason why option's maturity plays a role can be explained as follows. The default free option's value is higher, the longer the option maturity. A higher initial (at-the-money) option value means a higher claim proportion in case of default. The higher the claim proportion the bigger the "unfair" claim portion when the option is out-of-the-money at default, and hence the pattern.

bond holders. In other words, the bond will have a bigger credit spread under the default barrier distribution rule. Again, under the same distribution rule, it can not be determined ex ante which default barrier specification is preferred.

The simulation results, which are omitted for brevity, confirm the above general predictions. In addition, we find that the option is the least vulnerable when both the default barrier and the distribution rule are based on the market value of the option. It is the most vulnerable when the barrier is based on the initial option value yet the distribution rule is based on the market value. Intuitively, this is because the option portion of the default barrier is static and provides only partial protection, and yet the distribution implies that the option holder will never receive more than the market value of the option upon default.

As for credit spreads, although the general predictions are confirmed, when the option is at-the-money, or when the debt ratio is not very high, the differences in spreads among the alternative covenant and payoff specifications are generally not discernible. The difference in spreads becomes large only when, for example, the liability from the option's position is large (with in-the-money options).

Overall, the simulation results show that under general conditions, the two default barrier specifications do not lead to very different valuations for vulnerable options and the defaultable bonds. For vulnerable options, the best specification is one where both the default barrier and the distribution rule are based on the market value of the option. For defaultable bond, the lowest spread is associated with a default barrier based on the market value of the option, but a market value based distribution rule. Among the four barrier-distribution combinations, some are more plausible than others. The insights from our analysis help determine how a covenant should be set up properly.

4. Conclusion

Most of the existing studies on credit risk treat valuations of defaultable bonds and vulnerable options in separation. There are two main drawbacks. Firstly, the possibility of one type of liability going into default triggered by the other is totally ruled out, yet examples of such occurrence in practice are plenty. Barings Bank is a case in point. Secondly, it is far too unrealistic to assume, e.g., that bond holders have total claim against the firm in case of default. As derivatives become ever more prevalent in corporate treasuries, bond holders have found more and more accompanies as liability holders of the firm. The existing literature on vulnerable options has a drawback of its own. Most studies on this topic define default by comparing the asset value of the option-issuing firm with an exogenously specified default barrier, which usually takes the form of corporate debt. Option holders are fully paid off as long as the firm value is above the debt level. This flies in the face of both common sense and reality. Given that most derivative securities have theoretically infinite payoffs (such as a call), merely requiring the firm to be technically solvent with respective to regular debt does not guarantee full payoff to option holders. The end result of this erroneous assumption is the overestimation of the vulnerable option's value.

The current paper overcomes the aforementioned drawbacks by combining the two strands of literature. We introduce a second group of liability holders in the form of call option holders who are assumed to have equal claim priority as bond holders. The call option's maturity is assumed to be shorter than the bond's. In this framework, the default boundary is a sum of the minimum requirements imposed by bondholders and derivative holders. If default occurs before the option's maturity, then the two groups of liability holders will claim against the firm's assets according to a distribution rule either based on pre-specified requirements or based on market value of the two instruments. In our model, both the default barrier and the firm value experience downward jumps at the option's maturity. The former drops by the amount of the option holder's covenant requirement, while the latter drops by the amount of the intrinsic value of the option, which could be zero. It is shown that our framework contains many existing models as special cases, including Merton (1974), Black and Cox (1976), Johnson and Stulz (1987), Longstaff and Schwartz (1995), and Briys and de Varenne (1997).

To assess the full impact of the additional liability in the form of a short call, we examine two alternative default barrier specifications, one based on the option value at initiation, and the other based on the market value of an otherwise default-free option. The debt portion of the default barrier is always stochastic due to a stochastic interest rate. Under each default barrier specification, we in turn examine two alternative settlement rules, one where the payoff proportions are based on the barrier specification, and the other where the proportions are simply based on market value of the bond and the option.

Extensive simulations show that our model is capable of generating a variety of credit spread term structure shapes, including upward sloping, downward sloping, and humped. Importantly, it can generate sizable short term credit spreads which are impossible within the conventional diffusion setting. It is also found that unless the debt ratio is very high, or the option's position is large, the two different barrier specifications do not seem to produce very different valuations for the vulnerable option and the defaultable debt. With our parameters, it appears that the option is the least vulnerable when the default barrier is based on the market value of the option, and the claim proportions (upon default) are based on the barrier specifications (as opposed to simply market value of the two instruments); the debt is the least risky under the above default barrier but a claim rule whereby the proportions are strictly based on market values of the two liabilities.

For vulnerable options, we have an interesting and unique finding. Under a particular covenant and payoff specification (i.e. when both the default barrier and the claim rule are based on the initial value of the option), a vulnerable option can be worth more than its default-free counterpart or its counterpart which is not subject to early default — a vulnerable option needn't be always vulnerable after all! This seemingly counter-intuitive result is due to the way we specify the default barrier and the claim rule. The default barrier is specified at the time the two types of liabilities are initiated and vary subsequently only due to interest rate fluctuations. Although this setup suits the bond holder well, it does not take into account the market value change of the option due to fluctuations in the optioned stock price. As a result, sometimes the option holder can receive more than the fair value of the option at default. Unless the default barrier and the claim proportion are both market value based, the above possibility always exists. This has profound implications in terms of fair valuation of defaultable options.

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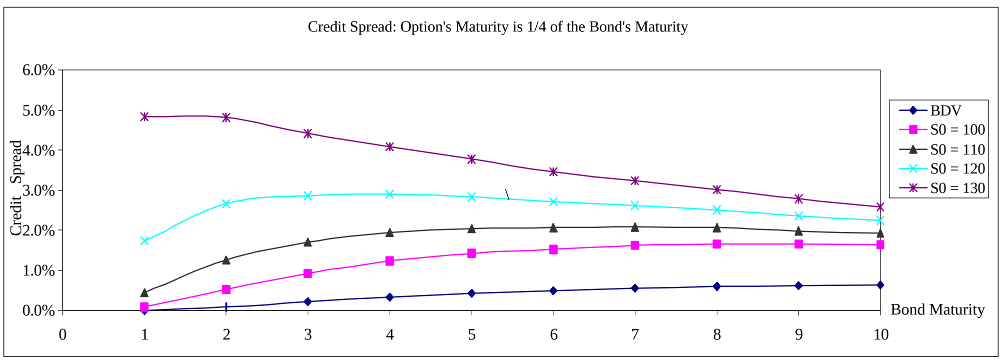
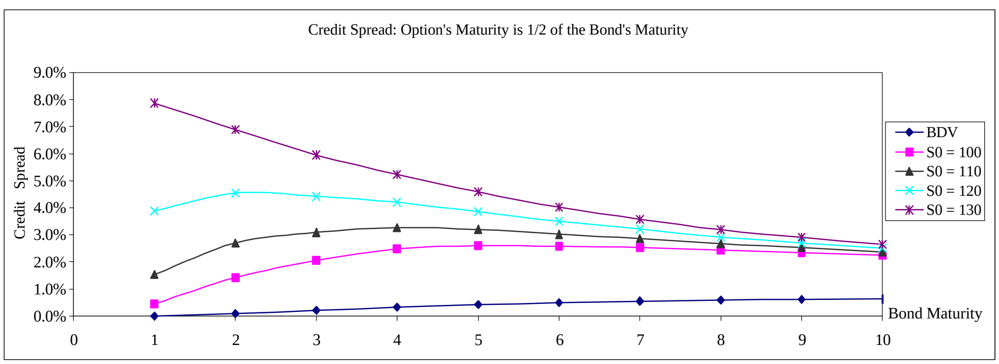
Appendices

Exhibit 1: Vulnerable Options and Term Structure of Credit Spreads

Bond Maturity (years)	Bond Value	Call Value	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
Panel A: S_0 = 100 , K = 100, C_{df} = 9.5323, C_{js} = 9.5176						
1	49.7759	9.5002	40.7300	9.99%	0.45%	0.00%
2	49.4872	9.4992	41.0200	9.98%	0.52%	0.08%
3	49.0633	9.4983	41.4400	9.96%	0.63%	0.21%
4	48.5807	9.4975	41.9300	9.94%	0.72%	0.33%
5	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
10	45.9594	9.4948	44.5500	9.81%	0.85%	0.63%
Panel B: S_0 = 120 , K = 100, C_{df} = 25.6847, C_{js} = 25.6441						
1	48.0989	25.1400	26.7700	9.99%	3.88%	0.00%
2	47.4139	25.1327	27.4600	9.98%	2.66%	0.08%
3	46.7585	25.1264	28.1200	9.96%	2.23%	0.21%
4	46.1598	25.1211	28.7300	9.94%	2.00%	0.33%
5	45.6224	25.1165	29.2700	9.92%	1.83%	0.42%
10	43.6634	25.1024	31.2400	9.81%	1.36%	0.63%





Unless otherwise specified, all results are computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. Headings and notation: "Spread_CW": spread generated from our model (Cao and Wei); "Spread_BDV": spread generated from the model by Briys and de Varenne (1997); C_{df}: default free call; C_{js}: vulnerable call based on Johson and Stulz (1987).

Exhibit 2: Effects of Capital Structure

Bond Maturity (years)	Bond Value	Call Value	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
Panel A: Debt Ratio = 0.3, $S_0 = K = 100$, $C_{df} = 9.5323$, $C_{js} = 9.5176$						
1	29.9863	9.5266	60.4900	9.99%	0.05%	0.00%
2	29.9777	9.5264	60.5000	9.98%	0.04%	0.00%
3	29.9596	9.5262	60.5200	9.96%	0.04%	0.00%
4	29.9249	9.5260	60.5500	9.94%	0.06%	0.01%
5	29.8694	9.5259	60.6100	9.92%	0.09%	0.02%
10	29.3388	9.5255	61.1400	9.81%	0.22%	0.14%
Panel B: Debt Ratio = 0.7, $S_0 = K = 100$, $C_{df} = 9.5323$, $C_{js} = 9.5176$						
1	67.5925	9.4383	23.0600	9.99%	3.50%	0.75%
2	65.8745	9.4297	24.7900	9.98%	3.04%	1.36%
3	64.4841	9.4227	26.1900	9.96%	2.74%	1.56%
4	63.3781	9.4171	27.3100	9.94%	2.48%	1.60%
5	62.4759	9.4127	28.2100	9.92%	2.27%	1.58%
10	59.6752	9.4010	31.0400	9.81%	1.60%	1.30%

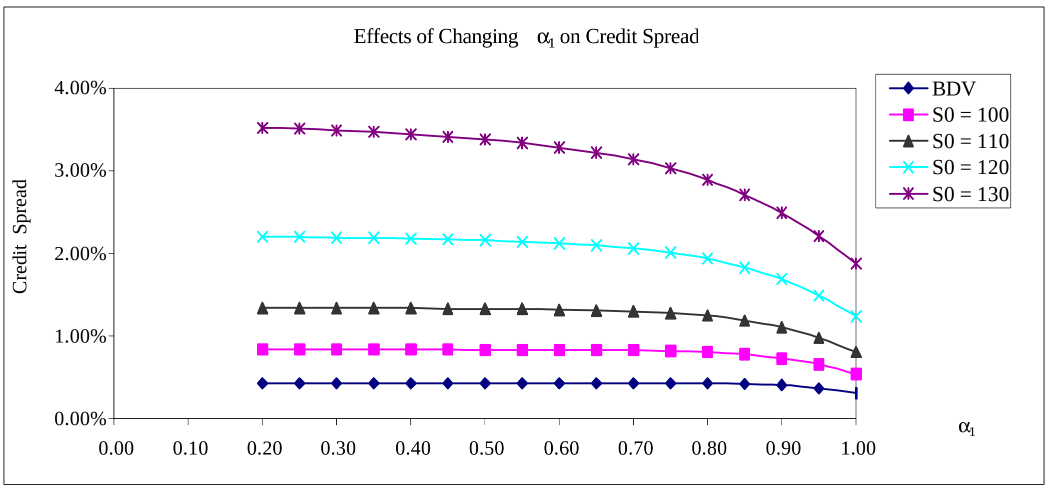
Panel C: Changing Debt Ratio while Fixing the Bond Maturity at 5 Years
$S_0 = K = 100$, $C_{df} = 9.5323$, $C_{js} = 9.5176$

Debt Ratio	Bond Value	Call Value	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
0.1	9.9992	9.5317	80.4700	9.92%	0.00%	0.00%
0.2	19.9869	9.5300	70.4900	9.92%	0.01%	0.00%
0.3	29.8694	9.5259	60.6100	9.92%	0.09%	0.02%
0.4	39.3612	9.5167	51.1300	9.92%	0.32%	0.14%
0.5	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
0.6	55.8054	9.4567	34.7500	9.92%	1.45%	0.91%
0.7	62.4759	9.4127	28.2100	9.92%	2.27%	1.58%
0.8	68.1251	9.5047	22.4200	9.92%	3.21%	2.36%
0.9	73.0600	9.8121	17.1700	9.92%	4.17%	3.20%

Unless otherwise specified, all results are computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%, a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25, \alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and $T_b = 5$.
Headings and notation: "Spread_CW": spread generated from our model (Cao and Wei); "Spread_BDV": spread generated from the model by Briys and de Varenne (1997); C_{df}: default free call; C_{js}: vulnerable call based on Johson and Stulz (1987).

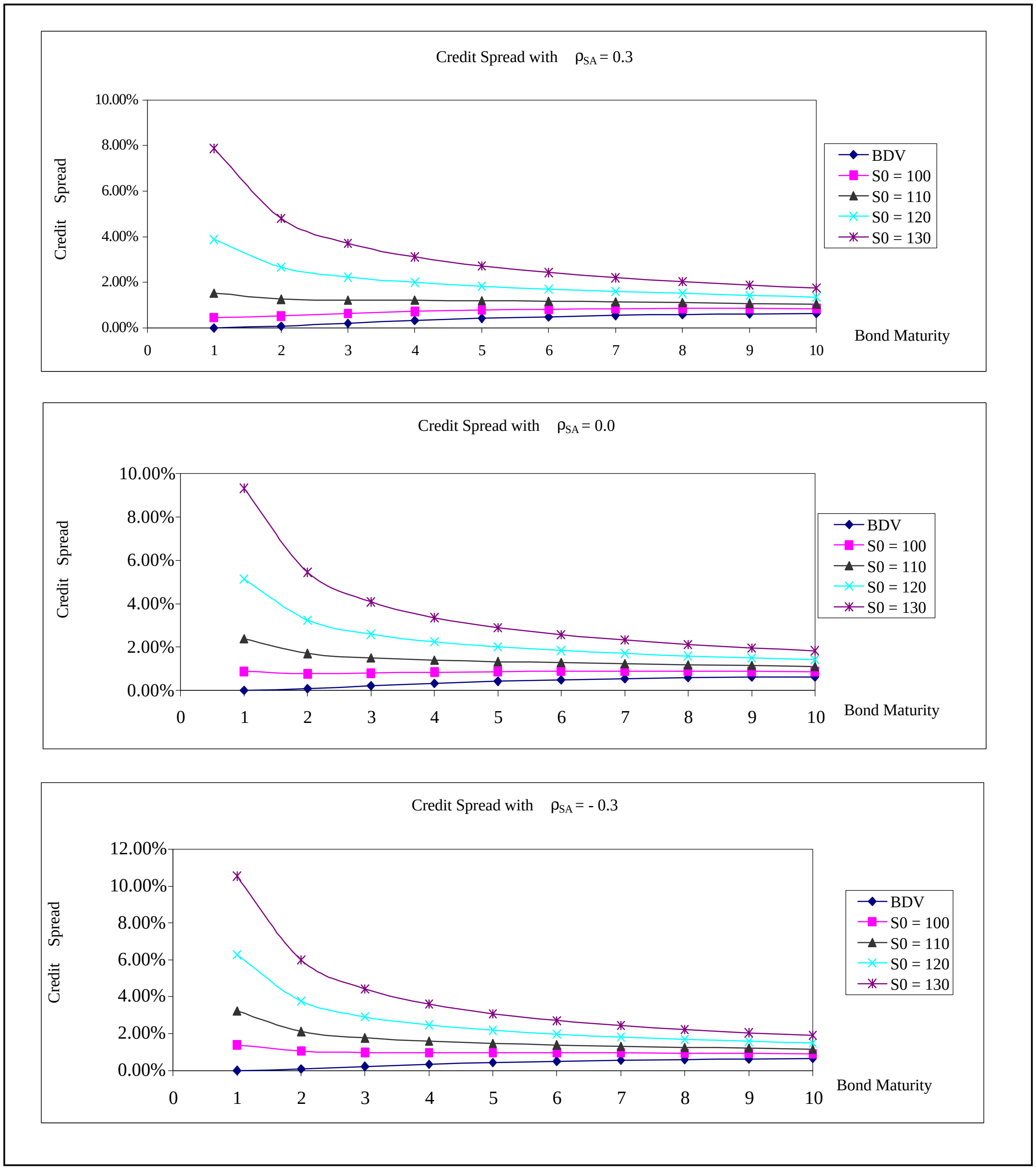
Exhibit 3: Degree of Covenant Protection (α_1)

α_1	Bond Value	Call Value	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
			S_0 = 100, K = 100, C_{df} = 9.5323, C_{js} = 9.5176			
0.20	47.9515	9.5315	42.5200	9.92%	0.84%	0.43%
0.25	47.9519	9.5311	42.5200	9.92%	0.84%	0.43%
0.30	47.9525	9.5305	42.5200	9.92%	0.84%	0.43%
0.35	47.9531	9.5298	42.5200	9.92%	0.84%	0.43%
0.40	47.9540	9.5288	42.5200	9.92%	0.84%	0.43%
0.45	47.9550	9.5277	42.5200	9.92%	0.84%	0.43%
0.50	47.9564	9.5263	42.5200	9.92%	0.83%	0.43%
0.55	47.9583	9.5243	42.5200	9.92%	0.83%	0.43%
0.60	47.9612	9.5218	42.5200	9.92%	0.83%	0.43%
0.65	47.9659	9.5189	42.5200	9.92%	0.83%	0.43%
0.70	47.9746	9.5151	42.5200	9.92%	0.83%	0.43%
0.75	47.9916	9.5103	42.5000	9.92%	0.82%	0.43%
0.80	48.0250	9.5044	42.4800	9.92%	0.81%	0.43%
0.85	48.0878	9.4969	42.4200	9.92%	0.78%	0.42%
0.90	48.1988	9.4878	42.3200	9.92%	0.73%	0.41%
0.95	48.3846	9.4762	42.1500	9.92%	0.66%	0.37%
1.00	48.6705	9.4617	41.8800	9.92%	0.54%	0.31%



Unless otherwise specified, all results are computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%, a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, $T_b = 5$, and a debt ratio of 0.5. Headings and notation: "Spread_CW": spread generated from our model (Cao and Wei); "Spread_BDV": spread generated from the model by Briys and de Varenne (1997); C_{df}: default free call; C_{js}: vulnerable call based on Johson and Stulz (1987).

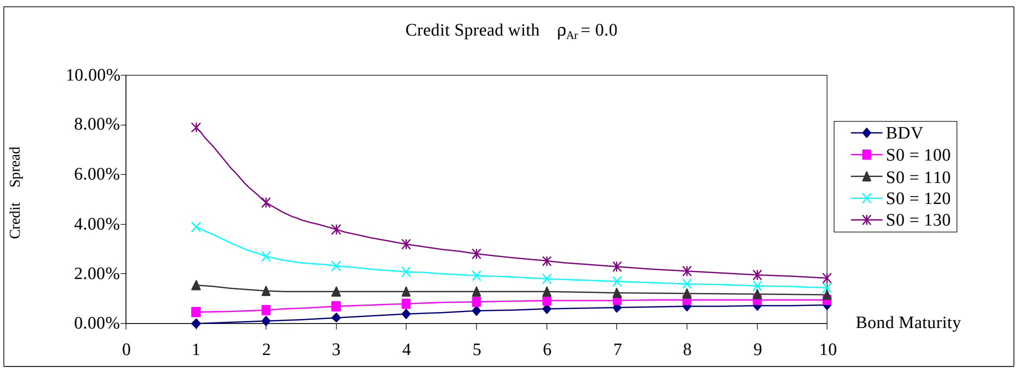
Exhibit 4: Effects of Correlation
between Firm Value and the Optioned Stock Price (ρ_{sA})

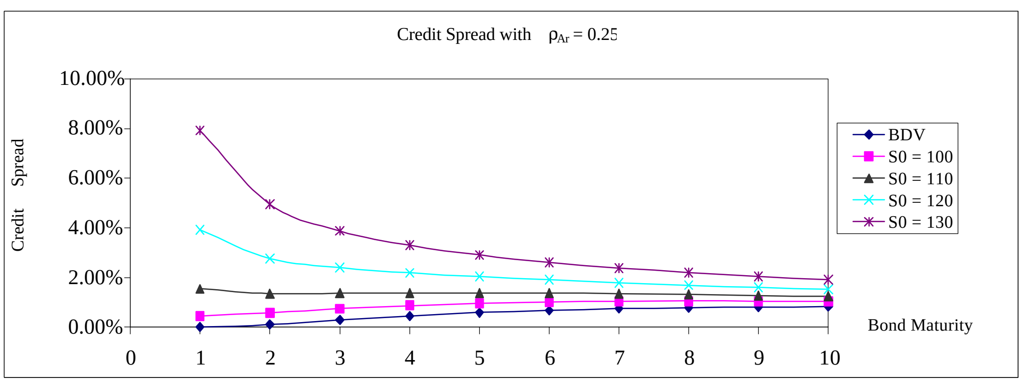


Unless otherwise specified, all results are computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{Ar} = -0.25$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. Legend: BDV: credit spread based on the model of Briys and de Varenne (1997).

Exhibit 5: Effects of Correlation
between Firm Value and Interest Rate (ρ_{Ar})

ρ_{Ar}	Bond Value	Call Value	Call Value, Cjs	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
		S_0 = 100,	K = 100,	C_{df} = 9.5323			
-0.55	48.3635	9.4983	9.5175	42.1400	9.92%	0.67%	0.33%
-0.45	48.2713	9.4978	9.5176	42.2400	9.92%	0.70%	0.36%
-0.35	48.1793	9.4974	9.5176	42.3300	9.92%	0.74%	0.39%
-0.25	48.0878	9.4969	9.5176	42.4200	9.92%	0.78%	0.42%
-0.15	47.9967	9.4964	9.5176	42.5100	9.92%	0.82%	0.46%
-0.05	47.9063	9.4960	9.5176	42.6000	9.92%	0.86%	0.49%
0.05	47.8166	9.4955	9.5176	42.6900	9.92%	0.89%	0.52%
0.15	47.7277	9.4950	9.5176	42.7800	9.92%	0.93%	0.56%
0.25	47.6396	9.4946	9.5176	42.8700	9.92%	0.97%	0.59%
0.35	47.5551	9.4943	9.5176	42.9700	9.92%	1.00%	0.62%
0.45	47.4689	9.4938	9.5177	43.0500	9.92%	1.04%	0.66%
0.55	47.3836	9.4933	9.5177	43.1400	9.92%	1.07%	0.69%





Unless otherwise specified, all results are computed with $S_0 = 100$, $K = 100$, $A_0 = 100$, $r_0 = 10\%$, $a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, $T_b = 5$, and a debt ratio of 0.5. Headings and notation: "Spread_CW": spread generated from our model (Cao and Wei); "Spread_BDV": spread generated from the model by Briys and de Varenne (1997); C_{df}: default free call; C_{js}: vulnerable call based on Johson and Stulz (1987).

Exhibit 6: A Closer Examination of Vulnerable Options

Option's Maturity (years)	Bond Value	Call Value	Call Value, Cjs	Default-free Call	Equity Value	Riskfree Yield	Credit Spread_CW	Credit Spread_BDV
				Panel A: S_0 = 90, K = 100				
0.50	48.6322	4.2216	4.2189	4.2272	47.1600	9.92%	0.55%	0.42%
1.00	48.0221	8.5912	8.5615	8.6084	43.4100	9.92%	0.81%	0.42%
1.50	47.1844	12.5047	12.4035	12.5706	40.3100	9.92%	1.16%	0.42%
2.00	46.0515	16.3657	15.8079	16.2384	37.4000	9.92%	1.65%	0.42%
2.50	45.0037	20.3369	18.8210	19.6751	34.5800	9.92%	2.11%	0.42%
3.00	44.0805	24.0737	21.4892	22.9171	32.1400	9.92%	2.52%	0.42%
3.50	43.1697	27.4735	23.8620	25.9878	29.4200	9.92%	2.94%	0.42%
4.00	42.4003	30.8444	25.9737	28.9038	27.0300	9.92%	3.30%	0.42%
4.50	41.6725	33.8648	27.8623	31.6775	24.4800	9.92%	3.64%	0.42%
5.00	41.0227	37.0710	29.5564	34.3191	21.9200	9.92%	3.96%	0.42%
				Panel B: S_0 = 100, K = 100				
0.50	48.0878	9.4969	9.5176	9.5323	42.4200	9.92%	0.78%	0.42%
1.00	47.1043	14.4461	14.7580	14.8497	38.3900	9.92%	1.19%	0.42%
1.50	46.0658	18.4858	19.0430	19.3946	35.4800	9.92%	1.64%	0.42%
2.00	44.8802	21.9583	22.6396	23.4925	32.9800	9.92%	2.16%	0.42%
2.50	43.8957	25.4712	25.6926	27.2698	30.5500	9.92%	2.60%	0.42%
3.00	43.1025	28.7068	28.3155	30.7919	28.5000	9.92%	2.97%	0.42%
3.50	42.3222	31.5660	30.5848	34.0983	26.1600	9.92%	3.33%	0.42%
4.00	41.6836	34.4460	32.5678	37.2159	24.1800	9.92%	3.64%	0.42%
4.50	41.0782	36.8955	34.3096	40.1640	22.0000	9.92%	3.93%	0.42%
5.00	40.5070	39.6592	35.8524	42.9576	19.8600	9.92%	4.21%	0.42%

Unless otherwise specified, all results are computed with $K = 100$, $A_0 = 100$, $r_0 = 10\%, a = 0.2$, $b = 0.1$, $\sigma_r = 0.02$, $\sigma_A = 0.2$, $\sigma_S = 0.25$, $\rho_{sr} = -0.4$, $\rho_{sA} = 0.3$, $\rho_{Ar} = -0.25, \alpha_1 = 0.85$, $\alpha_2 = 1.00$, $\gamma_1 = 0.90$, $\gamma_2 = 1.00$, $T_c = 0.5$, and a debt ratio of 0.5. Headings and notation: "Spread_CW": spread generated from our model (Cao and Wei); "Spread_BDV": spread generated from the model by Briys and de Varenne (1997); C_{js}: vulnerable call based on Johson and Stulz (1987).